Workshop: Grammars, Regular Expressions, and Finite-State Automata

Introductory Problems

These three problems are of increasing difficulty. Problems 1 and 2 may be written as multiple choice questions as indicated, while Problem 3 would be better suited to a free-response problem.

1. Consider the following method for creating strings of 1's. Begin with the symbol x, and apply the following replacement rules as many times as desired and in any order.

$$\begin{aligned} x &\mapsto 111x && (1) \\ x &\mapsto 11111x && (2) \\ x &\mapsto 111 && (3) \end{aligned}$$

The process ends when only ones remain. For example, we may obtain a string of 16 ones using

$$x \overset{2}{\mapsto} 11111x \overset{1}{\mapsto} 11111111x \overset{2}{\mapsto} 1111111111111x \overset{3}{\mapsto} 1111111111111111.$$

How many strings of one or more 1's CANNOT be obtained by applying these rules?

ALTERNATE MULTIPLE CHOICE PROBLEM:

How many strings of one or more 1's CANNOT be obtained by applying these rules?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 15

2. Consider the following method for creating strings of 0's and 1's. Begin with the symbol x, and apply the following replacement rules as many times as desired and in any order.

$$\begin{aligned} x &\mapsto 1 && (1) \\ x &\mapsto 0x && (2) \\ x &\mapsto 1y && (3) \\ y &\mapsto 0z && (4) \\ y &\mapsto 1x && (5) \\ z &\mapsto 0 && (6) \\ z &\mapsto 0y && (7) \\ z &\mapsto 1z && (8) \end{aligned}$$

The process ends when only 0's and 1's remain. For example,

$$x \overset{3}{\mapsto} 1y \overset{4}{\mapsto} 10z \overset{8}{\mapsto} 101z \overset{6}{\mapsto} 1010.$$

Thus, x is replaced by $1y$ using (3), and then the symbol y is replaced by $0z$ using (4), and so on, until the last occurrence of x, y, or z is replaced by a 0 or 1 as described by the rules.

Describe all binary strings which can be produced by these rules.

Alternate multiple choice problem:

Which of the following numbers, interpreted as a binary string, can be obtained using the above rules?

(A) 5^{2011} (B) 6^{2011} (C) 7^{2011} (D) 8^{2011} (E) 9^{2011}

3. Consider the following method for creating strings of 0's and 1's. Begin with the symbol a, and apply the following replacement rules as many times as desired and in any order.

$$
\begin{array}{ll}
a \mapsto 1a1 & (1) \\
a \mapsto 11a & (2) \\
a \mapsto 1 & (3) \\
11a1 \mapsto b0 & (4) \\
b \mapsto b0 & (5) \\
b000 \mapsto a & (6)
\end{array}
$$

The process ends when only 0's and 1's remain. For example, the string 11 may be obtained as follows:

$$a \overset{1}{\mapsto} 1a1 \overset{2}{\mapsto} 111a1 \overset{4}{\mapsto} 1b0 \overset{5}{\mapsto} 1b00 \overset{5}{\mapsto} 1b000 \overset{6}{\mapsto} 1a \overset{3}{\mapsto} 11.$$

Describe all binary strings obtained by these rules.