

Ice Cream!!!!

For the solution to this problem and the contents of today's presentation, please visit www.vincematsko.com and click on Riga 2010.

Click on IMSA, then APS to see the course website for Advanced Problem Solving. You will find a selection of Problem Sets there.

WARNING: Your head may explode if you try to solve this problem.

Problem solving isn't just for IMSA students! Six logical friends – Arlen, Boris, Cecil, Doran, Edith, and Frank – having just finished some pretty hairy logic problems, decided to go to Wacky Jack's Primo Ice Cream Emporium to celebrate. Each ordered their ice cream in a waffle cone or a dish, except for the one who ordered the Slurp-A-Licious Bucket-O-Yum – a delectable concoction with two scoops each of Jack's five Specialty Flavors – in which case it came in a commemorative Wacky Jack Bucket (which you got to keep if you ate it all). From the following clues, can you decide who ordered what? NOTE: All solvers made their selections from Jack's five Specialty Flavors: Blueberry Farkle, Chocolate Smudge, Marshmallow Whoop, Pecan Passion, and Peppermint Blast. For the cones, you must specify the order of the scoops.

NOTE: The rest of this page is blank so that all the clues fit on the back of this page.

1. No cone had more than three scoops of ice cream, and no dish had more than five.
2. Except for the problem solver who got the Slurp-A-Licious Bucket-O-Yum, no one got more than one scoop of the same flavor.
3. The number of scoops in Edith's dish was the same as the sum of the numbers of scoops in Arlen's and Cecil's cones.
4. Cecil and Edith together had the same number of scoops as Arlen and Frank together.
5. If Frank had an even number of scoops, then there were 23 scoops among the six solvers.
6. If Frank had an odd number of scoops, then there were 25 scoops among the six solvers.
7. If Boris had an odd number of scoops, then Edith had an even number.
8. Boris had fewer scoops than Doran.
9. Exactly two solvers had exactly three scoops of ice cream.
10. Frank ordered a dish if and only if Boris ordered a cone.
11. Arlen had three scoops of ice cream if and only if no two cones had the same number of scoops.
12. Exactly four solvers had Marshmallow Whoop, exactly four had Pecan Passion, and exactly two had both.
13. Either Arlen's flavors were a subset of Edith's, or Frank's were a subset of Cecil's.
14. Frank did not order Pecan Passion, and Cecil did not order Marshmallow Whoop.
15. If Cecil ordered just one scoop, then Arlen ordered Pecan Passion.
16. Boris and Doran were the only two to order Peppermint Blast.
17. Arlen and Frank had no flavors in common if and only if Boris and Frank had exactly one flavor in common.
18. Two cones had a scoop of Chocolate Smudge second from the top.
19. Frank had exactly two scoops if and only if Pecan Passion was always the bottom scoop of the cone it was on.

Solution: Let the letters A , B , etc., represent the number of scoops consumed by Arlen, Boris, etc. From (3) and (4), we have $E = A + C$ and $C + E = A + F$, from which we conclude that $F = 2C$. Then Frank had an even number of scoops, and so by (5) there were 23 scoops all together.

From (1), clearly none of Arlen, Cecil, Edith, and Frank ordered the Slurp-A-Licious Bucket-O-Yum. From (8), it follows that Doran did – and ate all ten scoops of ice cream!

Now from (3) and the fact that F is even, we conclude that $A + C + D + E + F$ is even, so that B is odd. Then by (7), Edith cannot have had three scoops of ice cream. Cecil can't have three, or Frank would have six, contradicting (1). Since F is even and $D = 10$, this leaves $A = B = 3$ by (9).

Now from (3) and (10), it follows that there are three cones and two dishes; (10) implies that Frank and Boris got a cone and dish in some order. Then from (11) and (1), it follows that the cones have 1, 2, and 3 scoops. Since there are two solvers with three scoops and the total number of scoops ordered is 23, we see that the dishes had 3 and 4 scoops of ice cream. So since Arlen's cone had 3 scoops, Edith's dish must have had 4, and thus Cecil's cone had just one. Since Boris had 3 scoops, he must have had the other dish by (11), leaving Frank's cone with 2 scoops.

Now from (12), every solver must have had at least one of Marshmallow Whoop or Pecan Passion. Since Frank did not have Pecan Passion, he had Marshmallow Whoop by (14), and similarly, Cecil ordered Pecan Passion.

By (15), Arlen ordered Pecan Passion. By (13), Arlen's Flavors were a subset of Edith's (since Frank ordered more scoops than Cecil), so Edith also ordered Pecan Passion. This makes four solvers who ordered Pecan Passion (since Doran ordered *every* flavor), and so since every solver had at least one of Marshmallow Whoop and Pecan Passion, Boris must have ordered Marshmallow Whoop. Arlen cannot have ordered Marshmallow Whoop or else Edith would have also, making five solvers ordering Marshmallow Whoop. Since Cecil only ordered Pecan Passion, this leaves Edith as the fourth solver who ordered Marshmallow Whoop.

By (16), Boris also ordered Peppermint Blast. This leaves Edith's two remaining flavors to be Blueberry Farkle and Chocolate Smudge. Since Arlen didn't order Marshmallow Whoop, this leaves him with Blueberry Farkle and Chocolate Smudge from (13).

Now Frank's second flavor must be Chocolate Smudge by (18). Since he has a flavor in common with Arlen and with Boris, it follows from (17) that Boris and Frank have more than one flavor in common, and so Boris must have ordered Chocolate Smudge as well.

Using these facts along with (19) and (18), it follows that the solvers ordered (cones listed from bottom flavor to top):

- Arlen's cone: Pecan Passion, Chocolate Smudge, Blueberry Farkle;
- Boris's dish: Chocolate Smudge, Marshmallow Whoop, Peppermint Blast;
- Cecil's cone: Pecan Passion;
- Doran's Slurp-A-Licious Bucket-O-Yum;
- Edith's dish: Blueberry Farkle, Chocolate Smudge, Marshmallow Whoop, Pecan Passion;
- Frank's cone: Chocolate Smudge, Marshmallow Whoop.