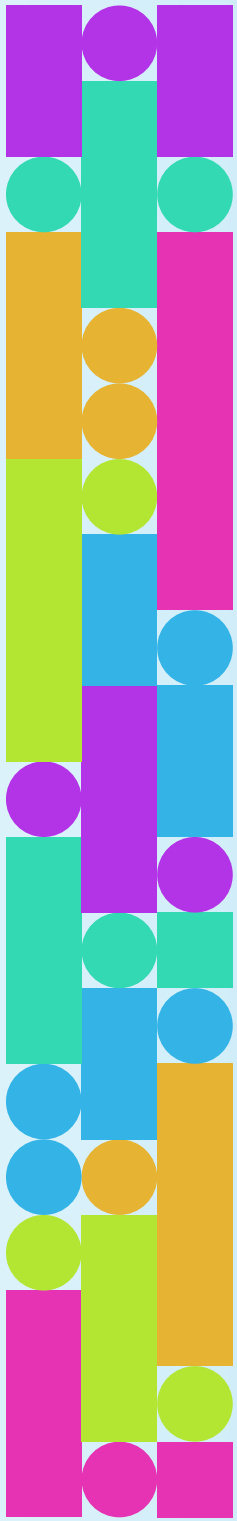

the role of student motivation in developing and assessing the acquisition of higher-order thinking skills

ICME-12
TSG-3
13 july 2012
seoul, korea

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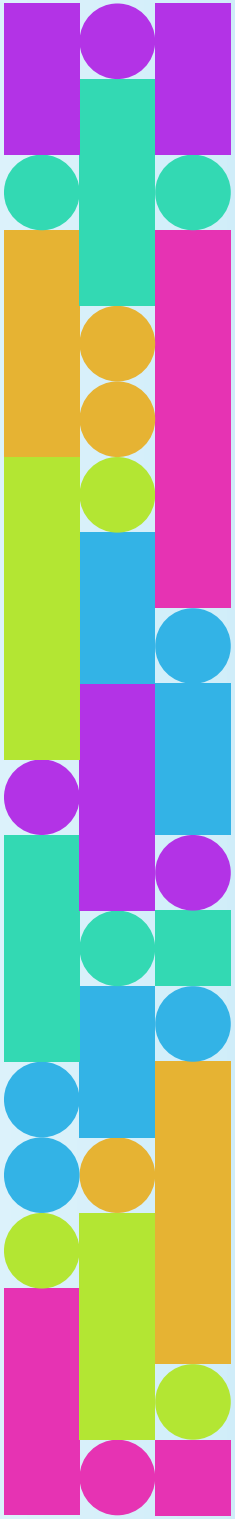


What is an Original Problem?

This assignment is a (possibly) short essay including the following components:

- Motivation
- Problem Statement
- Problem Solution
- Reflection

Motivation (LT):

I came up with this problem initially by writing the number “8” over and over again while thinking about this assignment. After a couple minutes of just repeating the same motion with my wrist, I realized that the “8” started to look like an ice skating rink, and thought about question 4 on Problem Set #3. Then I immediately tried to make a question similar to question 4, but with a different twist to it.

II. Problem Statement

The GC Auto Company specializes in manufacturing energy efficient automobiles. The motto of the company is "Green and Clean", but the company is worried because their manufacturing facility produces significant amounts of toxic waste. The company is trying to reduce the amount of toxic waste produced each year by using the following recursive formula to govern how much toxic waste is produced in the k'th year:

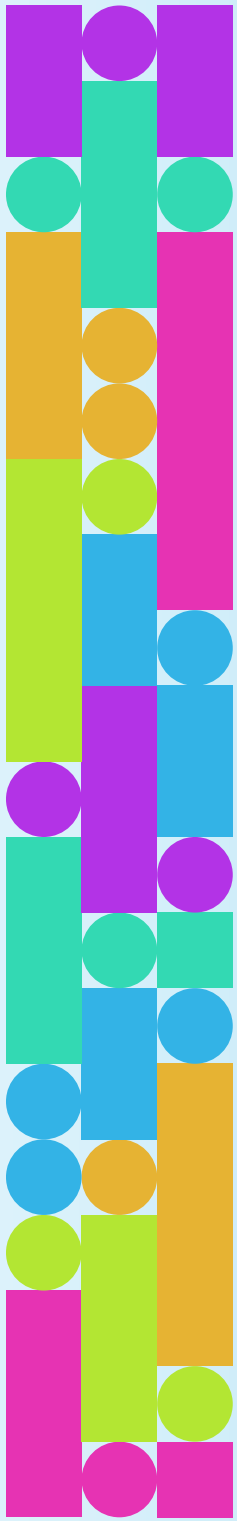
$$A_k = \frac{A_{k-1}}{2} + \frac{A_{k-2}}{4} \text{ for } k \geq 2$$

where A_k is the amount of toxic waste produced in the year k; $k = 0$ corresponds to the year 2010, $k = 1$ corresponds to the year 2011, and $k = n$ corresponds to the year $2010 + n$. The company produced 6000 kg of waste in 2010 ($A_0 = 6000$) and 5000 kg of waste in 2011 ($A_1 = 5000$).

The Environmental Agency has been keeping track of the total amount of toxic waste produced by GC Auto, and the EA requires the total amount produced since Jan 1, 2010 not to exceed 32,000 kg of toxic

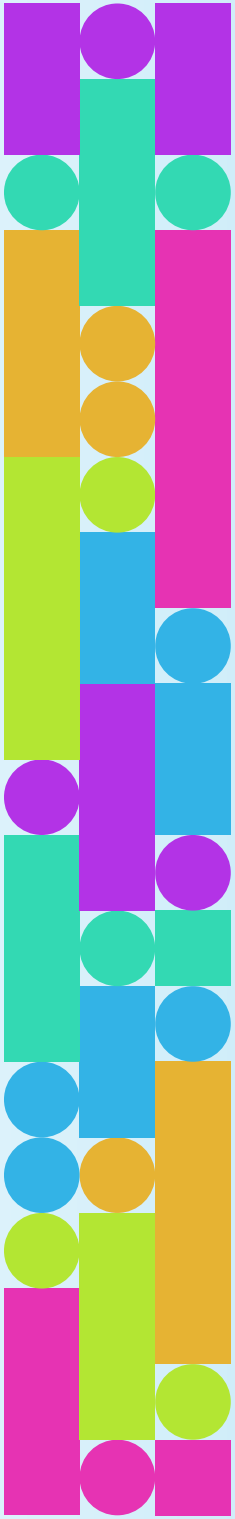
waste. The management at GC Auto is concerned because its waste-management formula (the recursive sequence) projects the company will produce 4000 kg of toxic waste in 2012. This means that by the end of 2012, the total waste of GC Auto will be 15,000 kg (since $A_0 = 6000$, $A_1 = 5000$, and $A_2 = 4000$).

The company is worried they might exceed the total waste limit of 32,000 kg at some time in the future. Will their total waste ever exceed 32,000 kg, provided that their annual waste follows the recursive sequence? Find your answer by hand, without using a calculator or computer.



Other examples (edited for brevity):

1. Using a graphical method, solve for a and b:
$$(1ab)_7 = (330)_a, \qquad (a42)_b = (31b)_8.$$
2. Describe and analyze the behavior of the graph of $y = \cos(\tan(x))$.
3. Find the period of $y = \sin^m(x)\cos^n(x)$ for positive integers m and n.
4. Describe graphs of the form $y = \sin(p\sin(qx))$.
5. Solve the equation $\sin(x)+1/2 = 0$. (Note: this was done by shifting the unit circle up $1/2$ along the y-axis and considering the geometry of the resulting circle.)



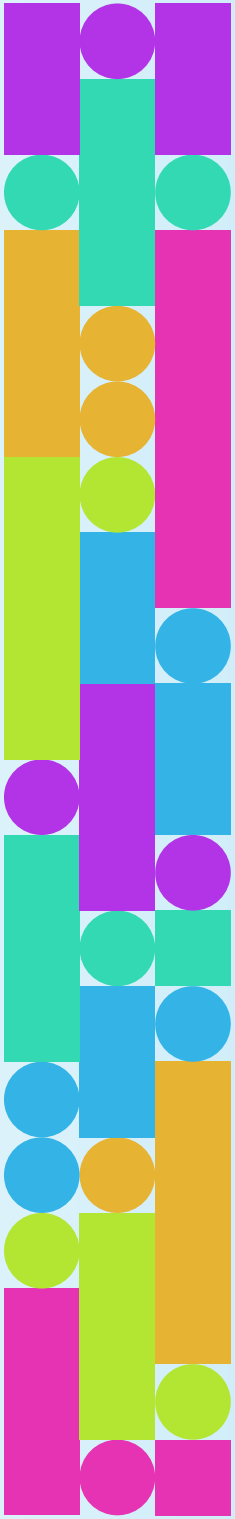
Other examples, continued:

6. Find the exact value of

$$\sum_{k=1}^{90} \cos^2(k) + \sum_{k=1}^{22} \tan^2(2k) - \sum_{k=1}^{44} \frac{1}{\tan^2(2k)} + \sum_{k=1}^{22} \frac{1}{\tan^2(2k)}.$$

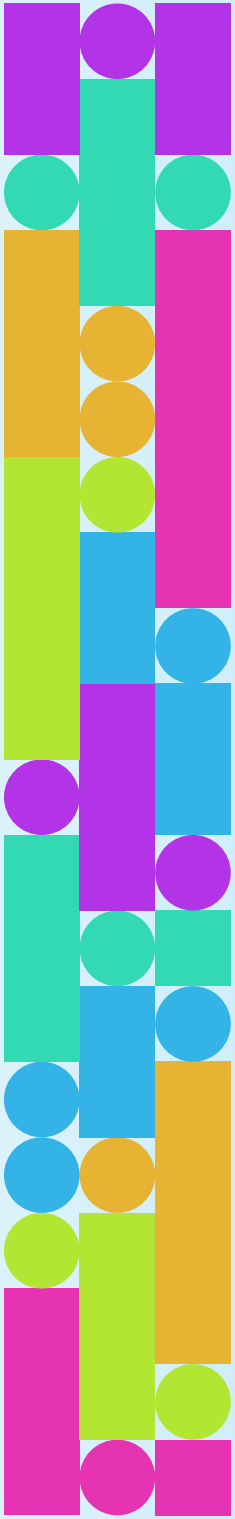
7. Solve the quartic equation $x^4 - 8x^3 + 25x^2 - 46x + 40 = 0$. (Note: This was done by finding a related cubic equation and using the method for solving cubics developed in the course Problem Sets.)

8. Suppose that a bishop and a knight are randomly placed on a chessboard. What is the probability that they attack each other?



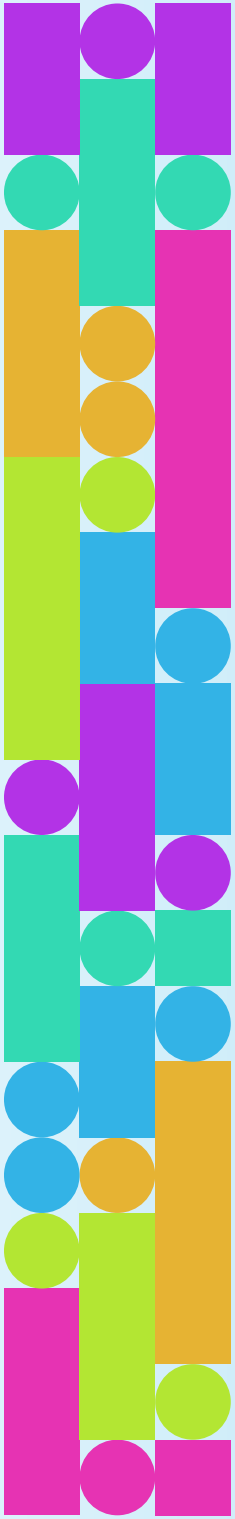
Reflection (YW):

Problem-writing was a very interesting and challenging experience this semester. I have never encountered anything like this before, so it was quite difficult for me to get started on writing my problem. I was not accustomed to thinking so conceptually about math, as before, I sort of treated math as problems that can be matched up with equations to obtain answers. I never really spent much time actually considering rules and laws and how/why they worked. This forced me to take on a new perspective that was fun and frustrating at the same time.



History of using Original Problems in class:

- Fall 2008, 2009: Advanced Problem Solving (self-selected)

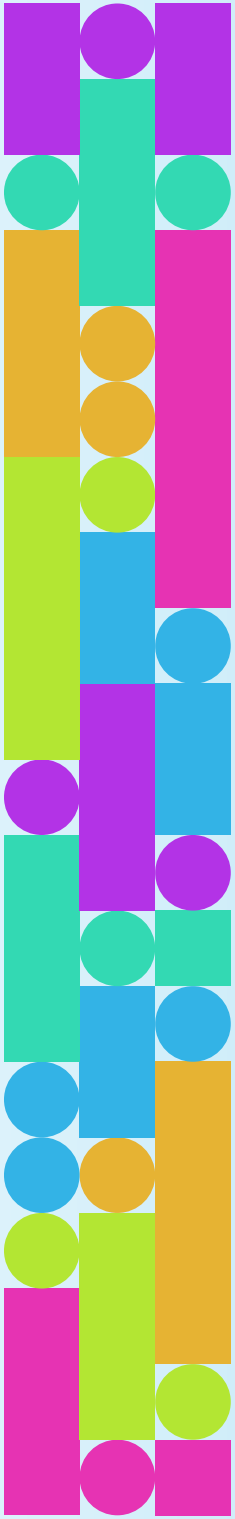


History of using Original Problems in class:

- Fall 2008, 2009: Advanced Problem Solving (self-selected)

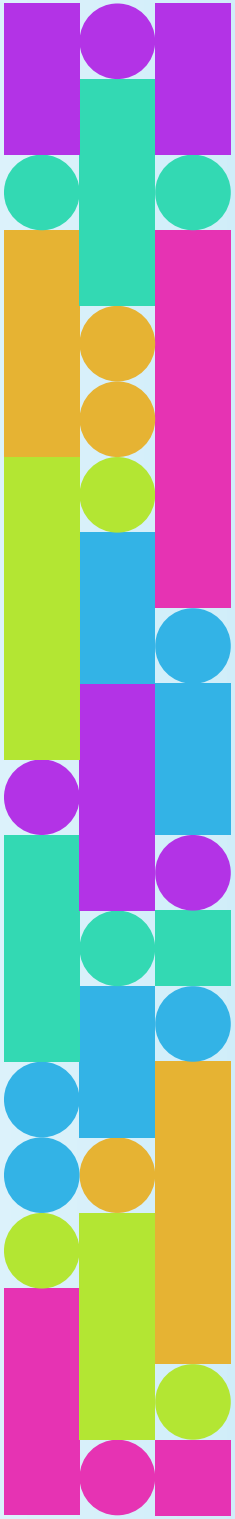
Student EL:

Anyone can write tedious, difficult problems that review core math subjects, but to write problems in a novel, challenging, and refreshing manner, one must be imaginative. I feel that this creative side of math is an often overlooked aspect of the field as many believe math to be an extremely black-and-white, rigid, and boring subject.



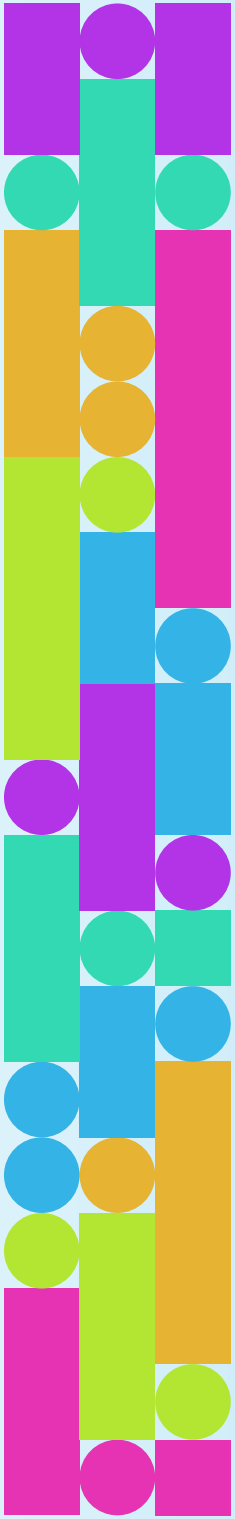
Student SA:

To be honest, I thought that the only thing necessary to write problems was being able to manipulate previously written problems enough to form an entirely new one. I really despise basing original problems off of previous problems, because I just don't think my work is authentic in that situation. I didn't know that I could just start drawing a bunch of lines and come up with an interesting problem. This was what was so surprising to me during the semester....I think creativity is key to writing original problems. I now bear witness to its lovely academic benefits.

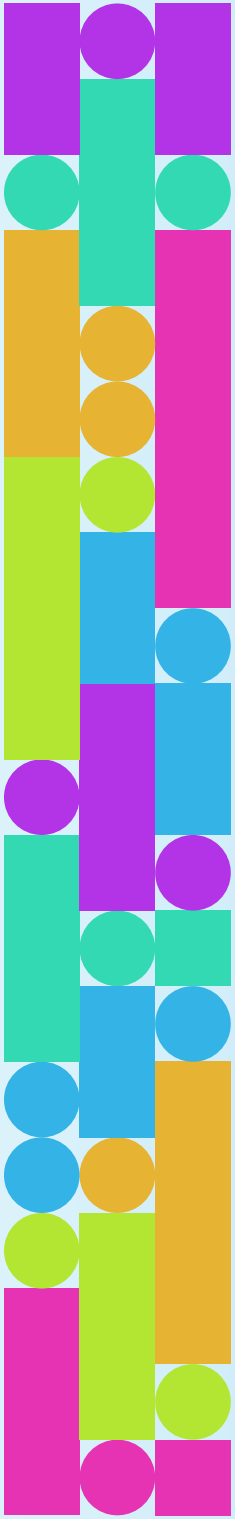


History of using Original Problems in class:

- Fall 2008, 2009: Advanced Problem Solving (self-selected)
- Spring 2010–Fall 2011: Honors BC Calculus
- Spring 2011: Traditional BC Calculus I
- Fall 2011: Precalculus (second of three semesters)
- Fall 2011: Traditional BC Calculus II
- Spring 2012: Traditional BC Calculus III

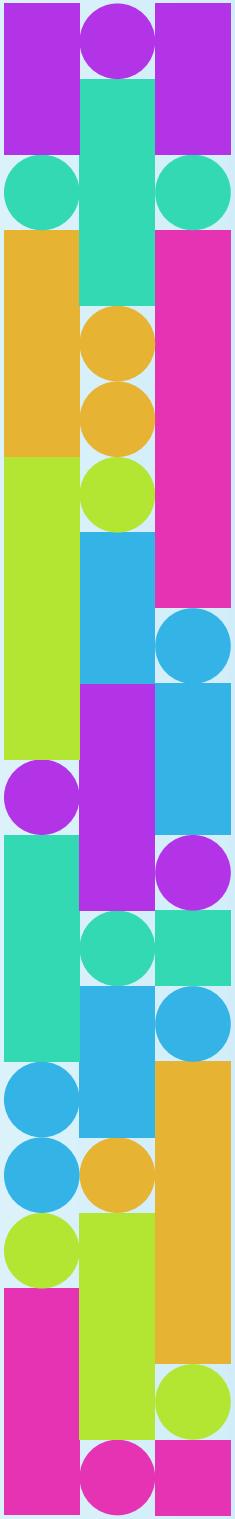


While there is general agreement of the importance of developing higher-order thinking skills, the attainment of such skills is difficult to assess. Students have reported that one particular type of assessment (the writing of original mathematics problems) is not only engaging, but has also stimulated the development of these skills. Undertaking a systematic study of the relationship between engagement, motivation, and the acquisition of higher-order thinking skills using the writing of original problems will provide insight into the nature of developing and assessing such skills. This will be accomplished by (1) giving students surveys about their problem-writing experiences after each assignment is due and (2) having students write brief reflective papers near the end of the semester.



Student subjects:

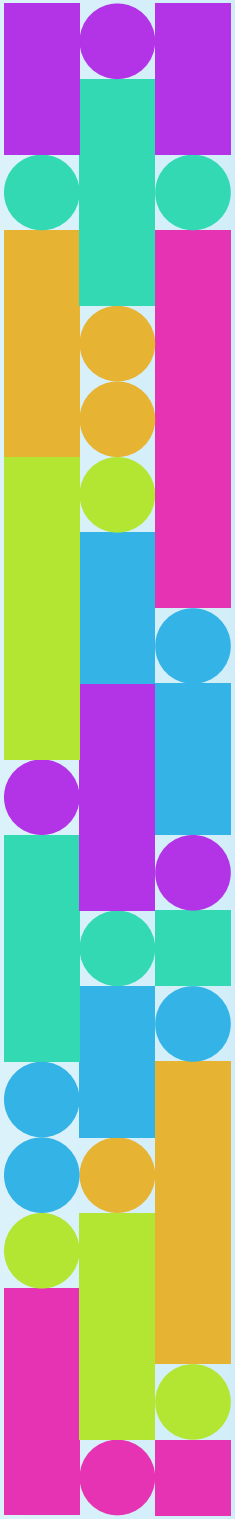
- 44 students in the Fall 2011 Mathematical Investigations III course (the third of four semesters in precalculus);
- 29 participants in 10th grade, 14 in 11th grade, 1 in 12th grade;
- No comparison subjects;
- Participation voluntary (all students participated).



Student participation:

- Write three Original Problems as part of the normal course assignments;
- Complete questionnaires after handing in the assignments (the last two of the three);
- Write a brief reflective paper after all assignments are done;
- Scan and compile all graded problems and the final reflection in one electronic document.

Also note the purpose of developing writing skills over the course of the semester.



The survey instrument included:

- 11 forced-choice questions (on a 1–5 Likert scale);
- 3 open-ended questions.

Examples of forced-choice questions:

1. I am more engaged and interested in mathematics when I am allowed to create my own problems.
2. The problems I choose to create I select because they are related to concepts I enjoy.
3. I find myself thinking about my original problems outside of class.

The open-ended questions were:

1. Has the exercise of creating original mathematics problems enhanced your motivation in math class? If so, in what ways?

2. Has the exercise of creating original mathematics problems enhanced your ability to think creatively? If so, in what ways?

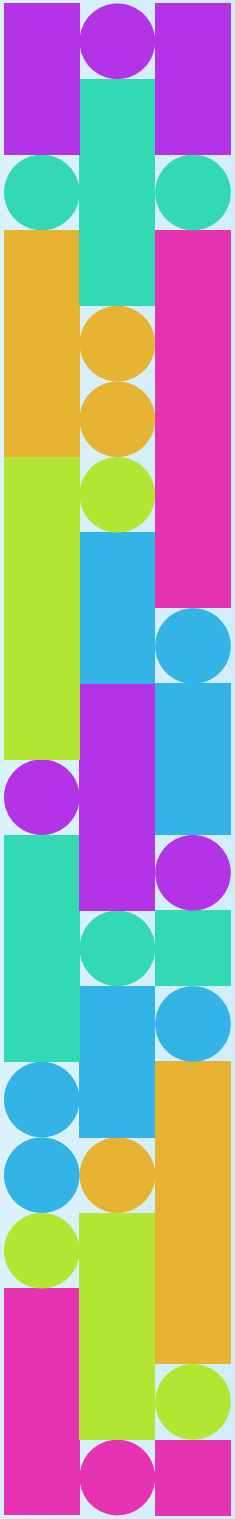
3. Have you been able to transfer any of the skills you have developed in the creation of original problems to other courses, including courses outside of mathematics? Give specific examples, if possible.

Main findings in analysis of Likert-scale data:

1. No gender-based differences – what might be an explanation for this?

2. The question with the highest mean (3.95) was, "I find creating problems more challenging than answering problems posed by the instructor;" the next highest mean was 3.51.

3. This mean of 3.51 was for the question, "Creating original problems causes me to think about my own thinking (metacognition) more."



Analysis of open-ended questions:

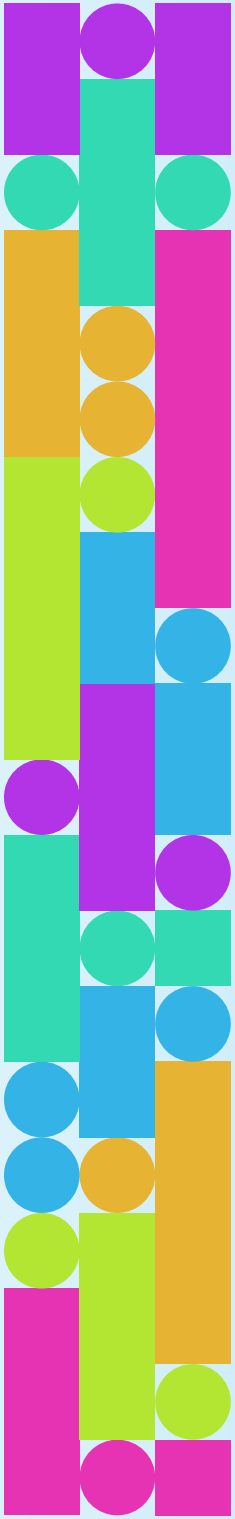
We were interested in whether students' autonomy in creating their own problems enhanced intrinsic motivation, which Lepper and Hodell (1989) suggest may be accomplished by:

1. presenting a challenging activity which enhances self-efficacy and feelings of competence;

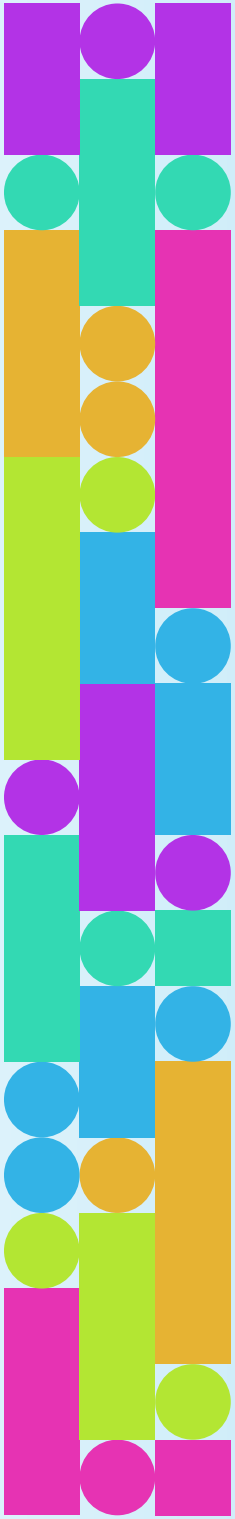
2. engaging curiosity about problems that seem complex or incongruous;

3. allowing students a sense of control and ownership over their own learning.

We found ample evidence of student engagement leading to an enhanced sense of efficacy, deeper engagement, and feelings of ownership and intrinsic motivation.



In several responses, we found that the challenge of problem creation leading to enhanced intrinsic motivation. We found that students used the exercise to challenge themselves beyond the concepts presented in the course. We also recognized in several responses that students had, indeed used the opportunity to take ownership for their learning, and, in one case, a shift from performance orientation (grades): "I am able to explore topics that interest me so I can care more about the solution. I have more motivation to solve the problem rather than just getting a good grade."

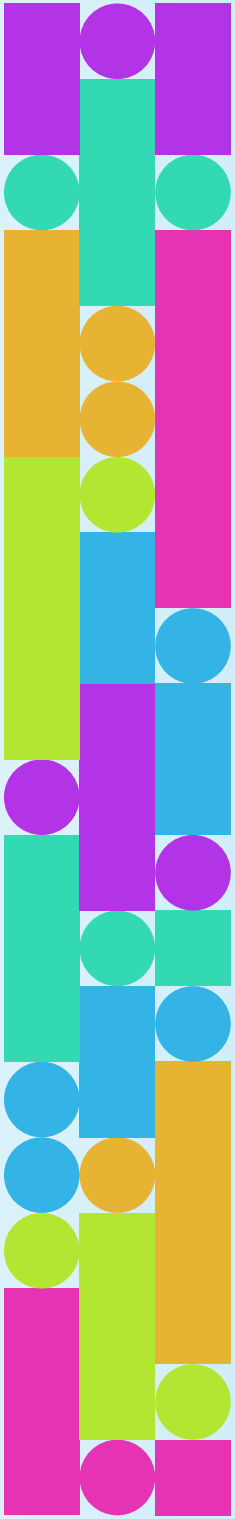


Students clearly indicated that the assignment enhanced their metacognition, although there was no prompt or probe in the open-ended questions asking specifically about metacognition:

- It has made me more aware of my problem solving skills.
- It has helped with my questioning skills.
- Some of my original problems were related to chemistry and physics, so I could apply it to other fields of study.
- I think it has taught me to think differently and more open-mindedly about the place of math in everyday life.

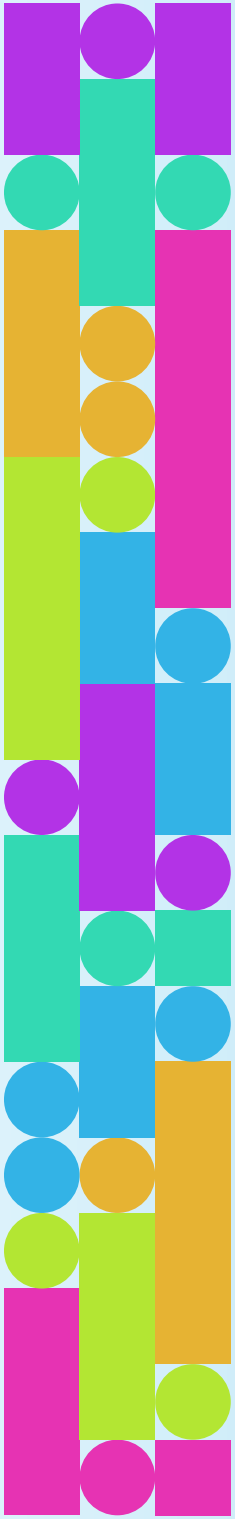
Summary:

First, when given the freedom to design their own learning experiences, students will make connections to events, experiences, and memories of deep significance to them. Second, although we were exploring motivation and creativity in math, we found that, without asking them to articulate it as such, students were able to suggest how the problem-creation exercise enhanced the development of their metacognitive strategies.



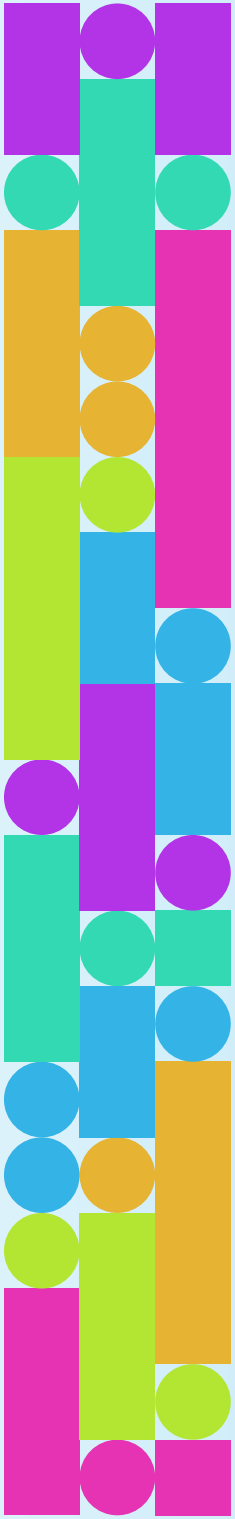
Further questions for future research:

- What would be the student response to problem creation in disciplines other than mathematics?
- How might such an approach be received by middle school students?
- Would the pedagogical approach for more traditional students be as effective with students who are not identified as gifted in mathematics?
- Would the use of the problem creation activity enhance creativity, motivation, and metacognition as measured by standardized instruments?
- Are the effects of enhancing these traits evident in subsequent math classes?



In your classroom:

- Start small!
- Tailor the assignment to your strengths and interests.
- Grade gently:
  - A: Good, conceptual problem; well-written.
  - B: Routine problems, or significant errors in the solution.
  - C: Clear lack of planning or effort.
- Comment as you see fit and have the time.



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- This slideshow;
- The research paper;
- The Original Problem assignment;
- An essay on mathematical creativity;
- Two essays on problem-writing;
- A link to the Riga 10 talks.