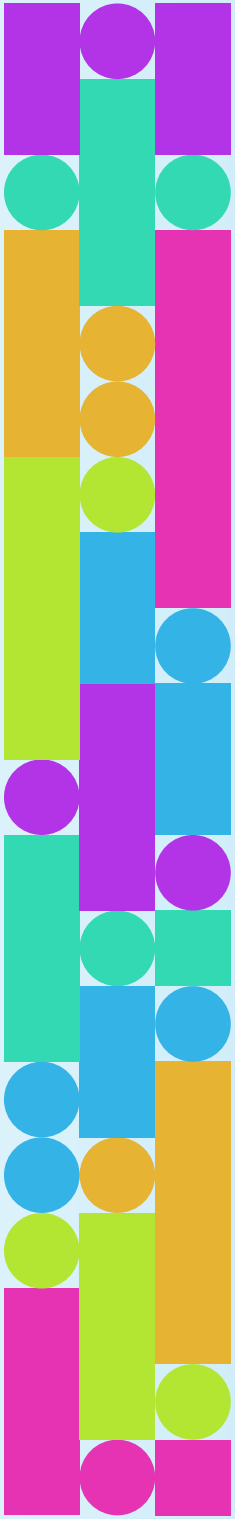
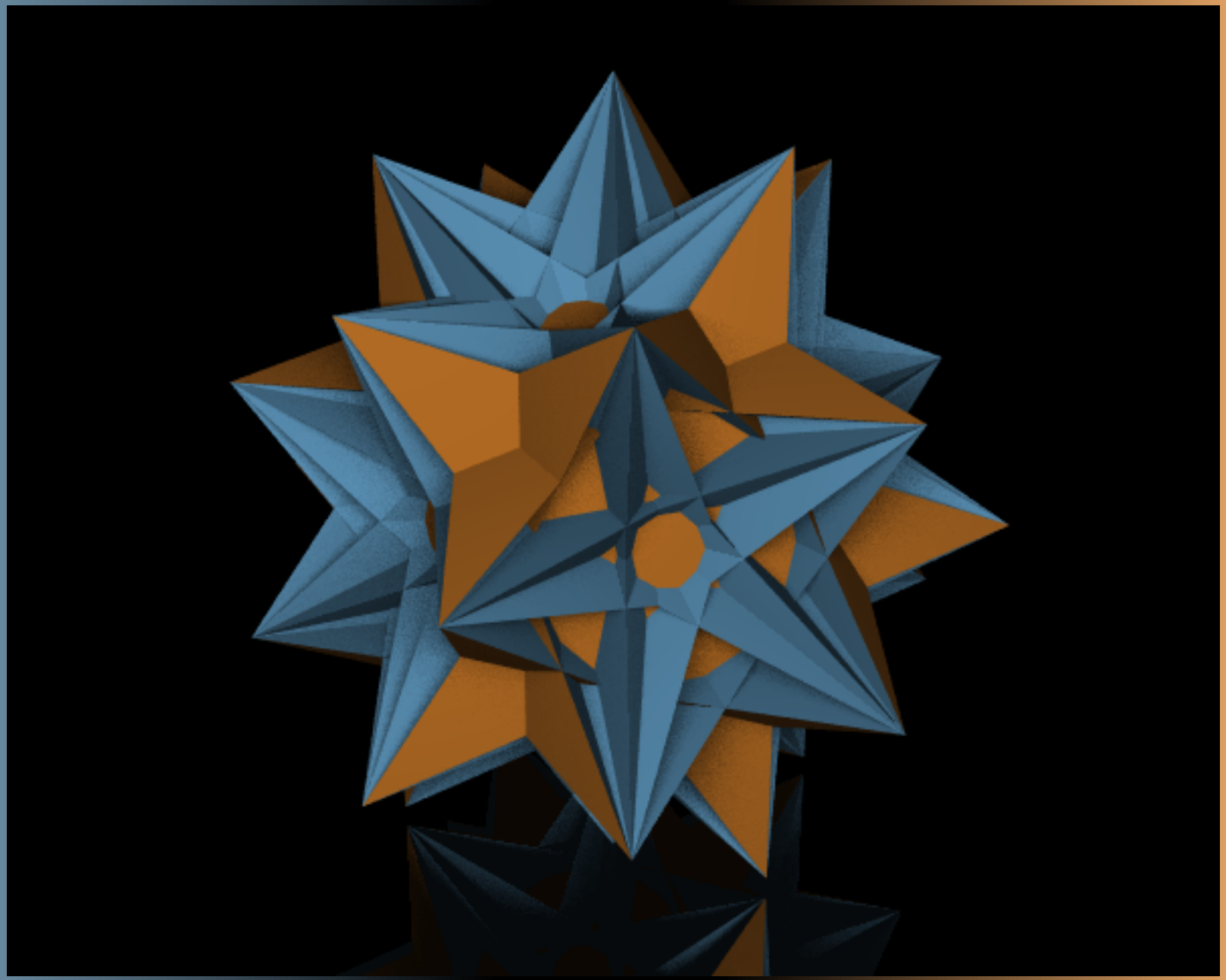
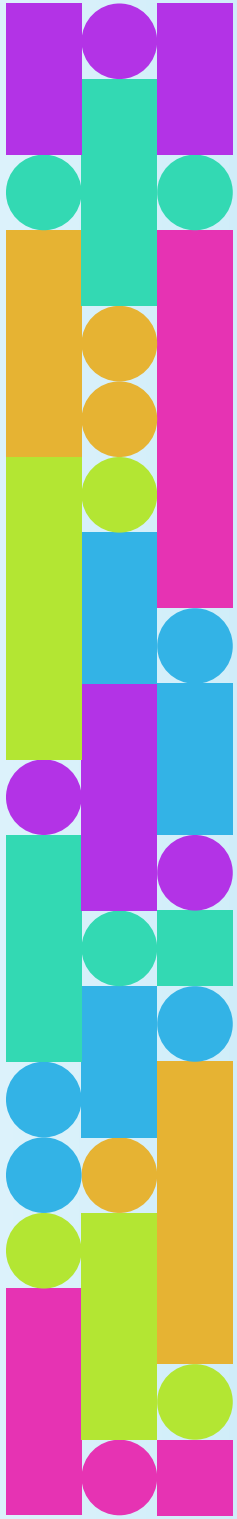


teaching mathematics as art

MCG-7
discussion group
12 july 2012
busan, korea

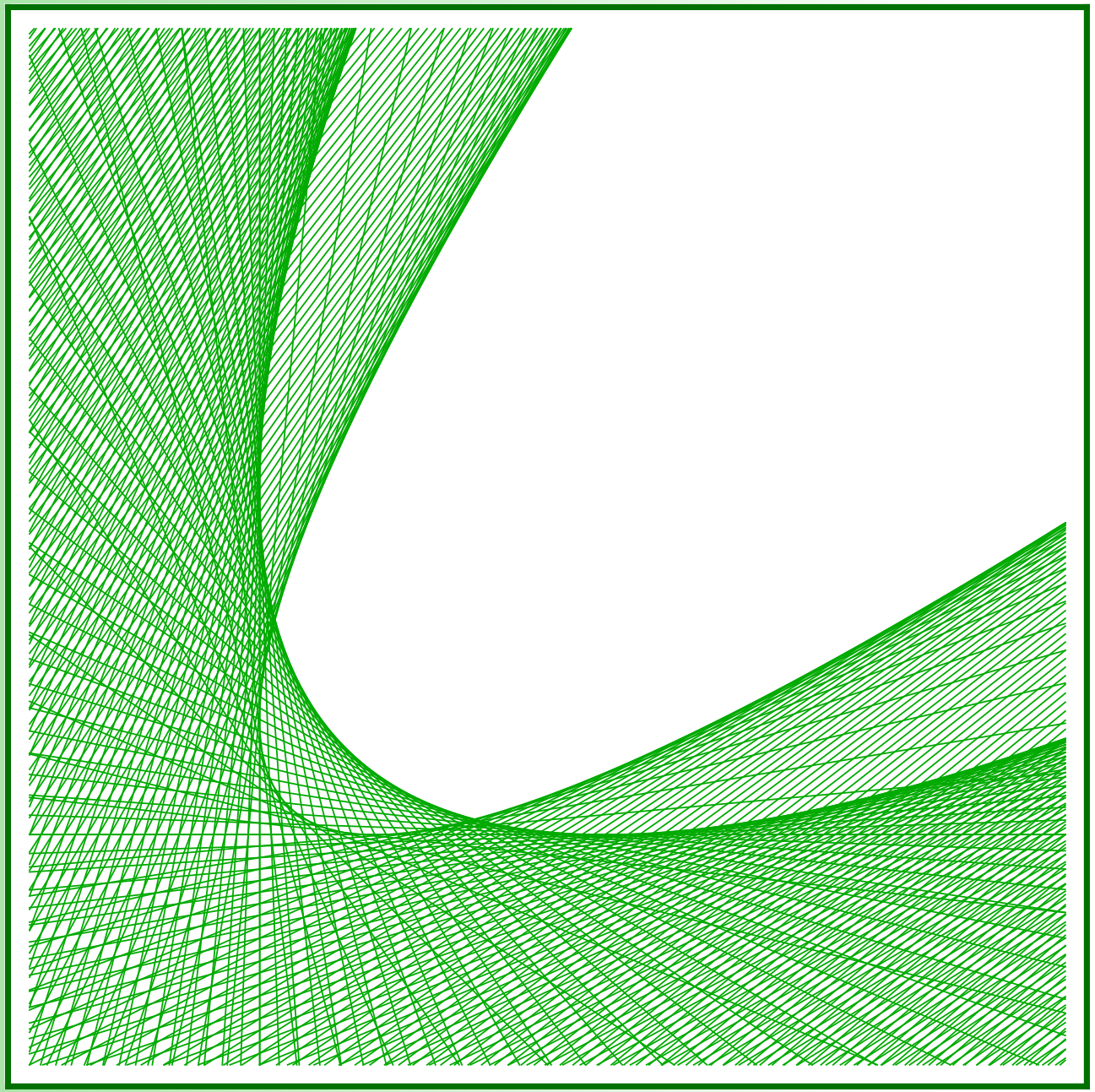
vince matsko, imsa

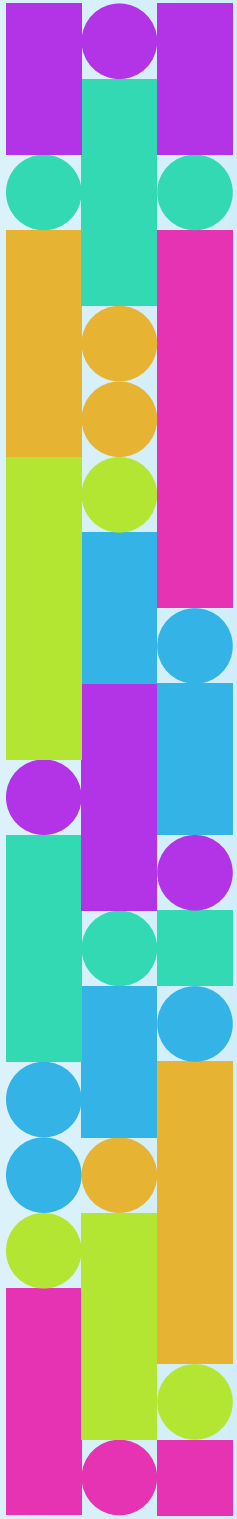




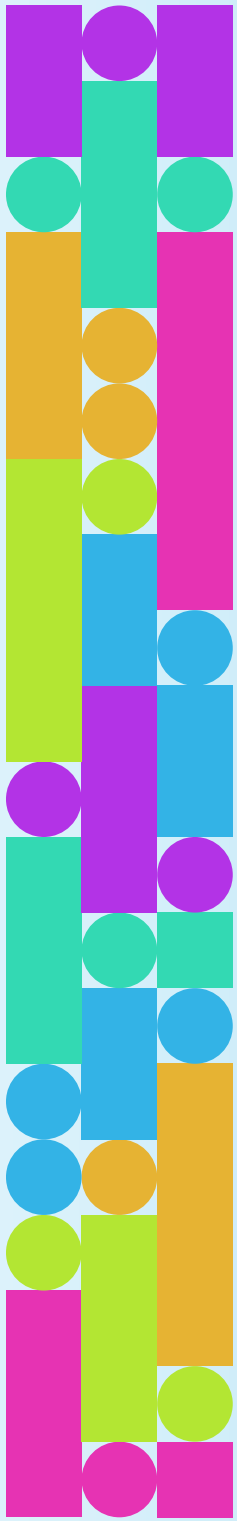
The first thing to understand is that mathematics is an art. The difference between math and the other arts, such as music and painting, is that our culture does not recognize it as such....Part of the problem is that nobody has the faintest idea what it is that mathematicians do.

–Paul Lockhart, in
A Mathematician's Lament
(as will be the case for all quotes)

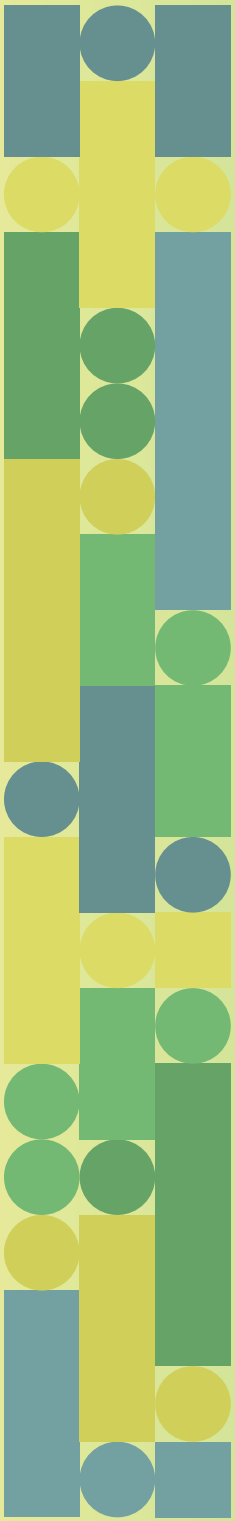


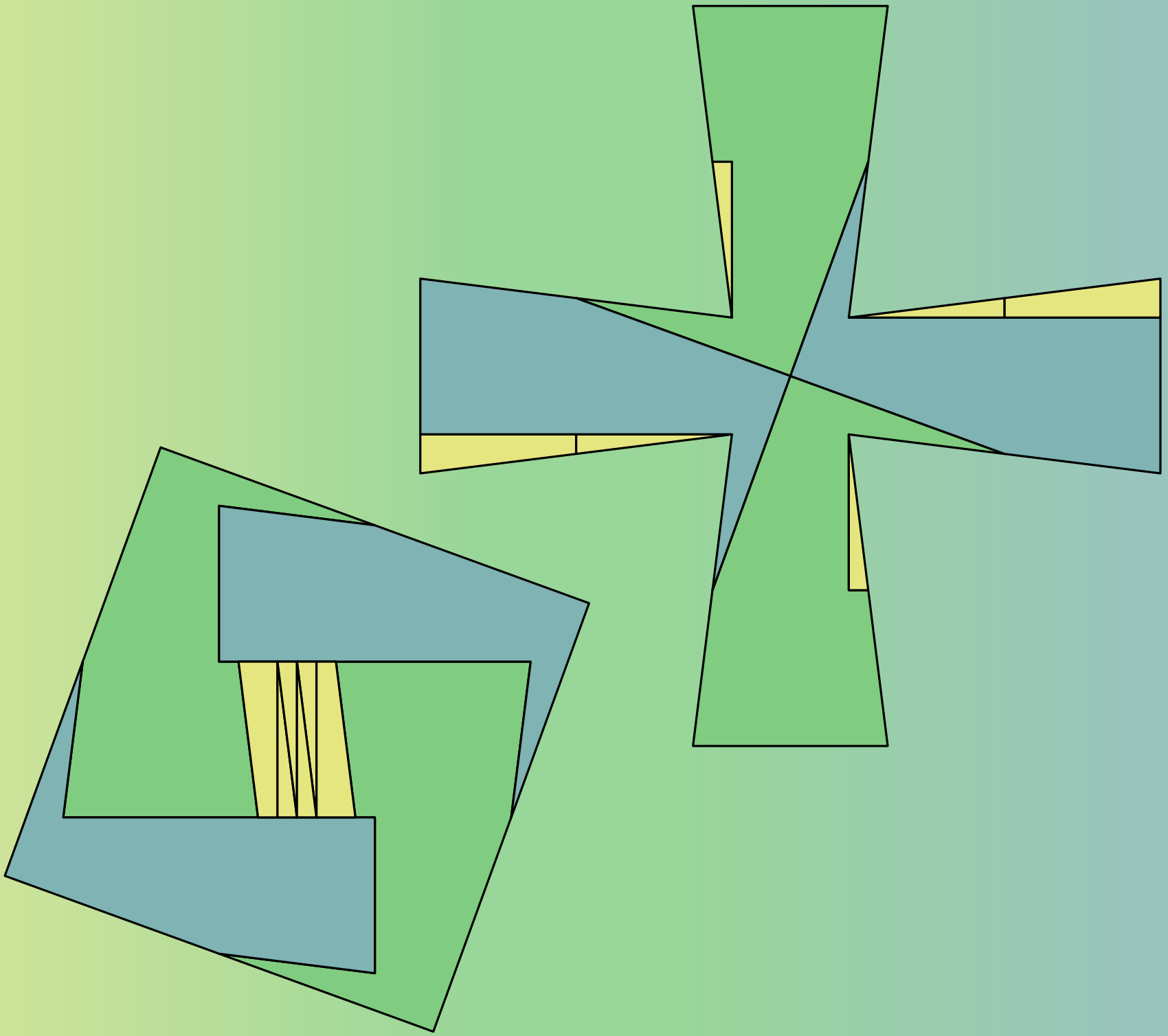


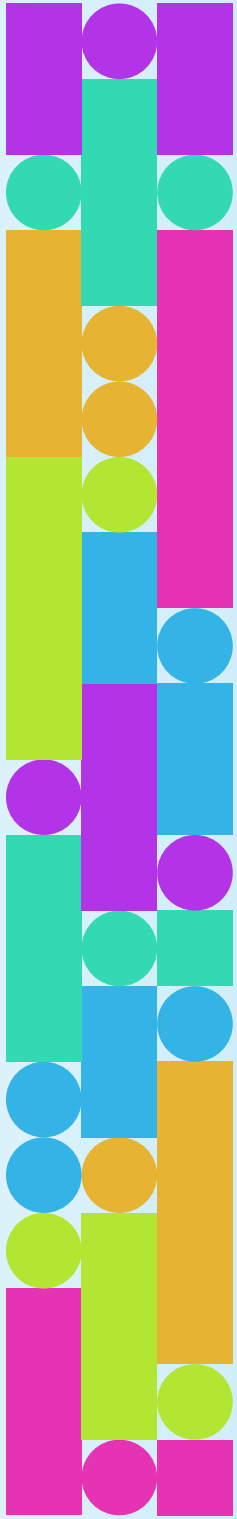
Mathematics is *the art of explanation*. If you deny students the opportunity to engage in this activity – to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs – you deny them mathematics itself.



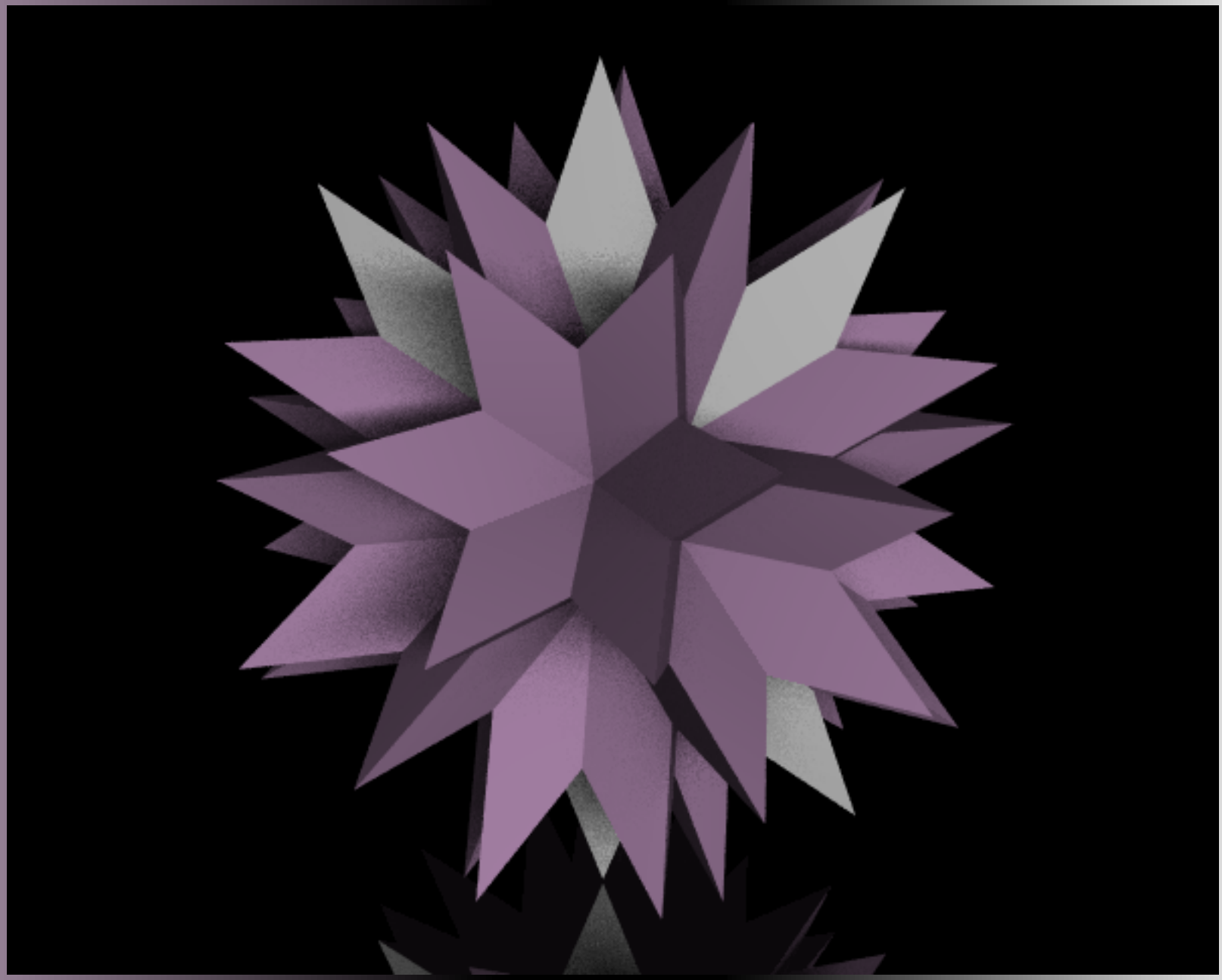
So no, I'm not complaining about the presence of facts and formulas in our mathematics classes. I'm complaining about the lack of *mathematics* in our mathematics classes.

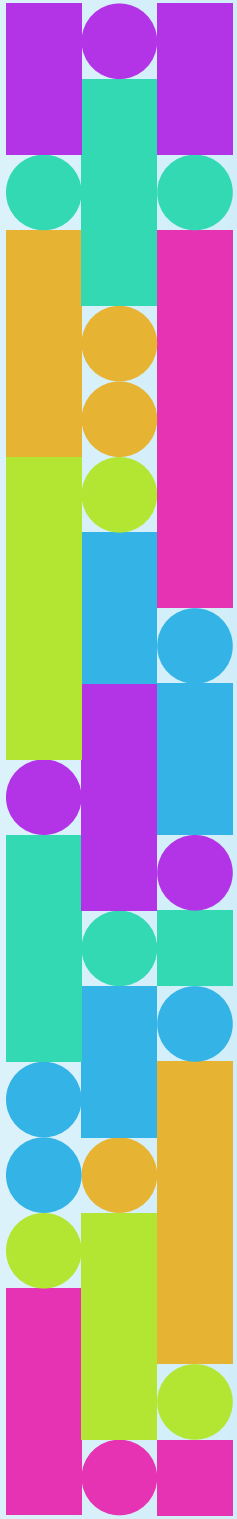




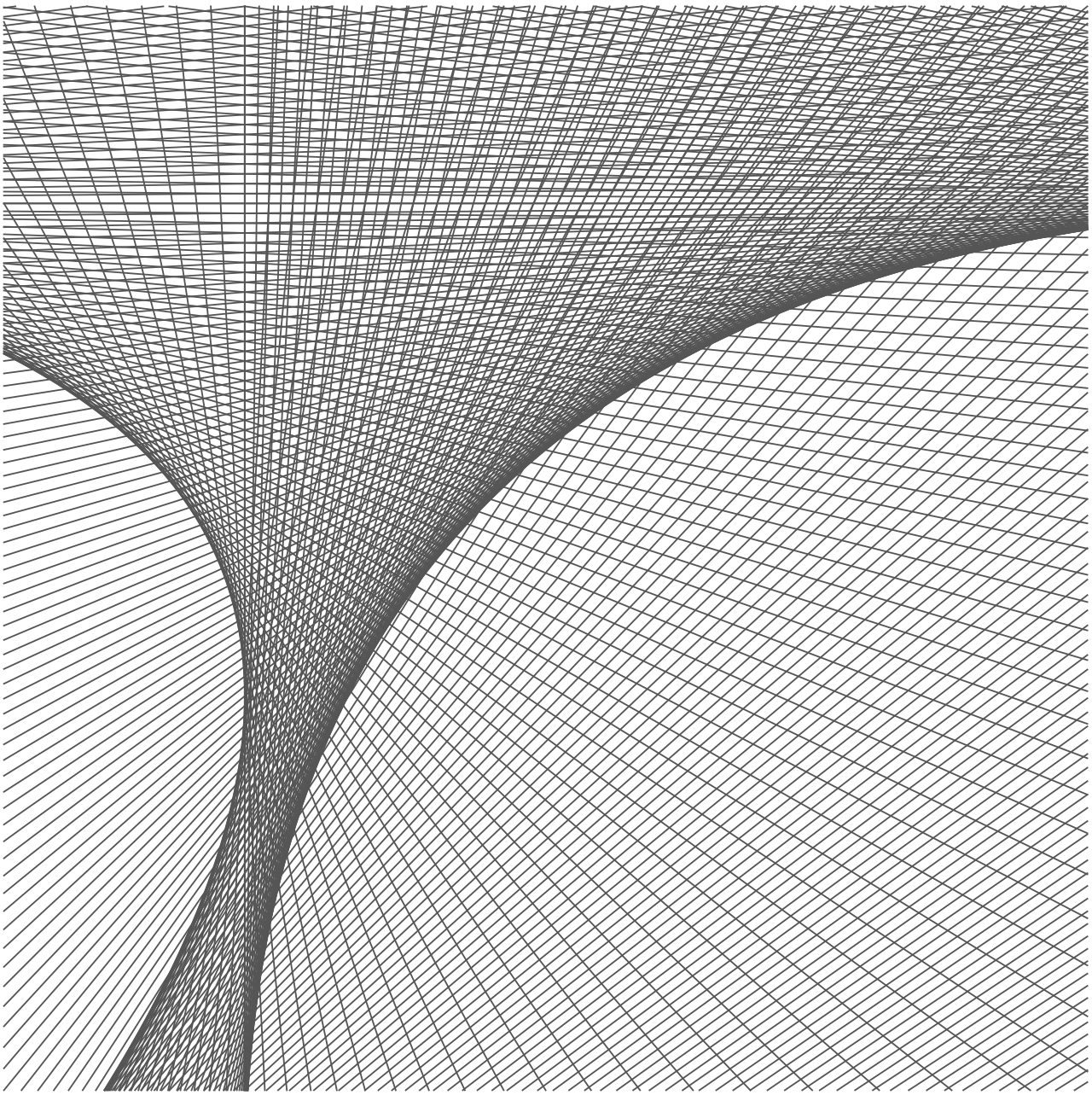


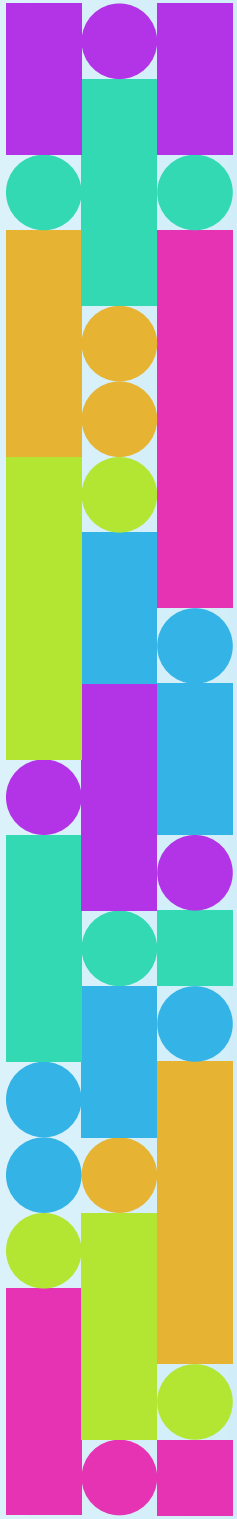
I can understand the idea of training students to master certain techniques....But not as an end in itself. Technique in mathematics, as in any art, should be learned in context. The great problems, their history, the creative process that is the proper setting. Give your students a good problem, let them struggle and get frustrated....Wait until they are dying for an idea, then give them some technique. But not too much.





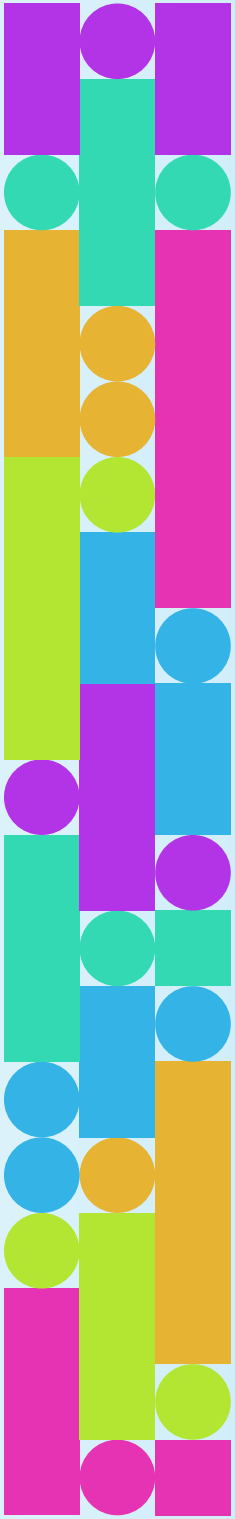
Why is it that we accept math teachers who have never produced an original piece of mathematics, know nothing of the history and philosophy of the subject, nothing about recent developments, nothing in fact beyond what they are expected to present to their unfortunate students?



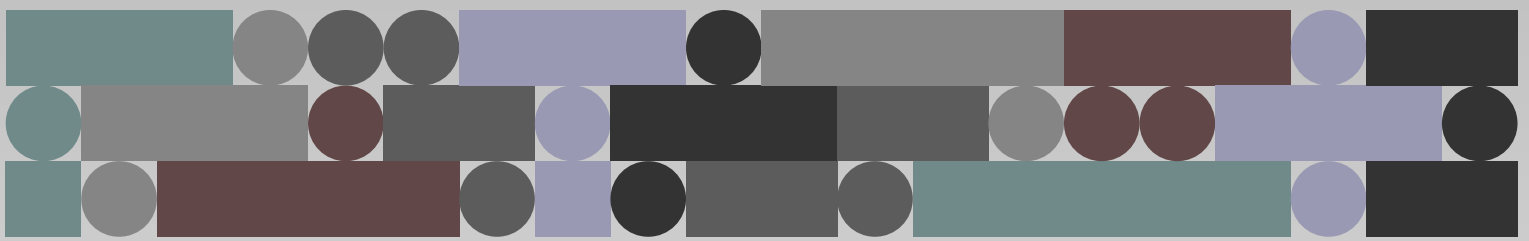


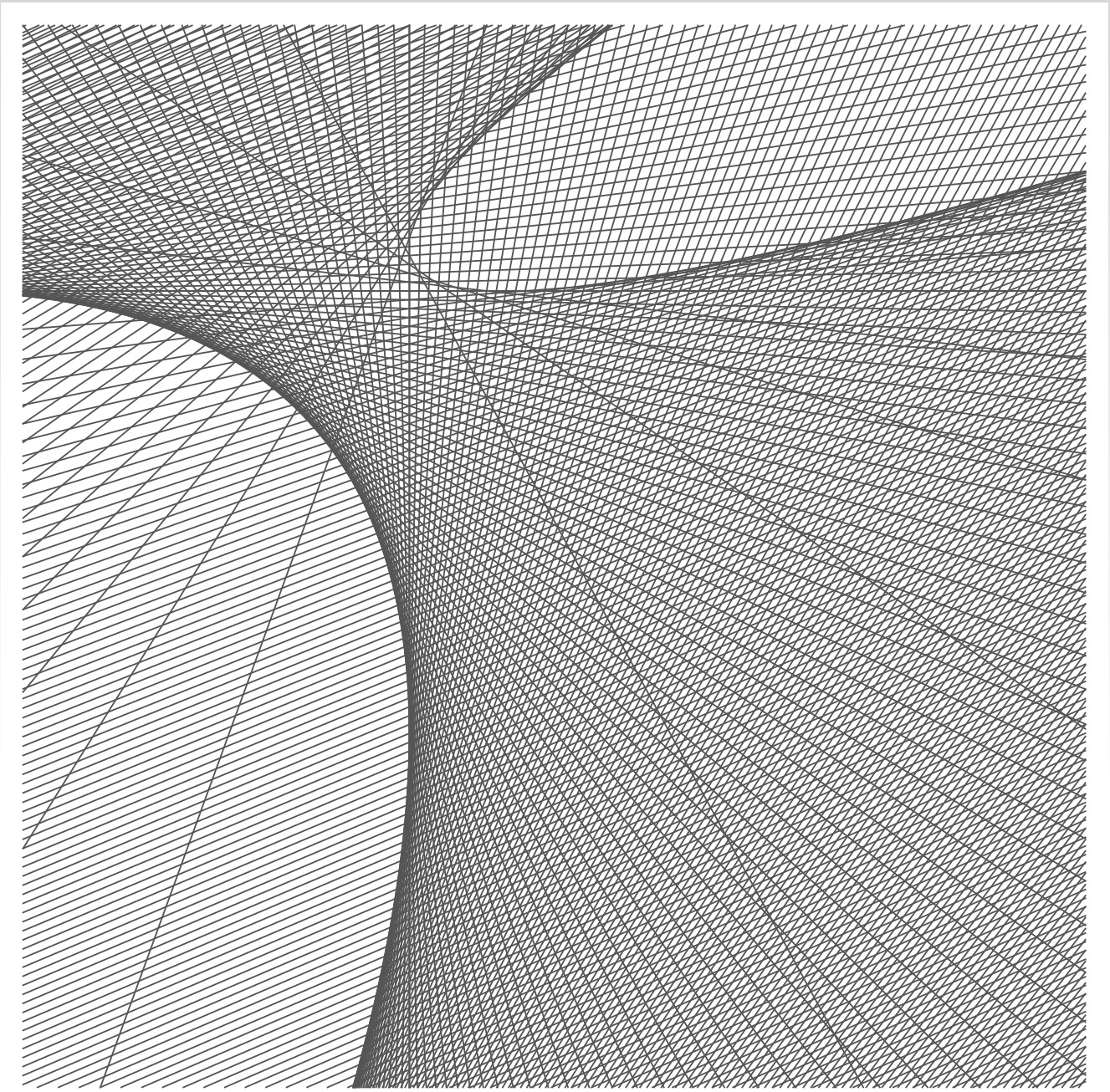
Teaching is not about information. It's about having an honest intellectual relationship with your students. It requires no method, no tools, and no training. Just the ability to be real. And if you can't be real, then you have no right to inflict yourself upon innocent children.

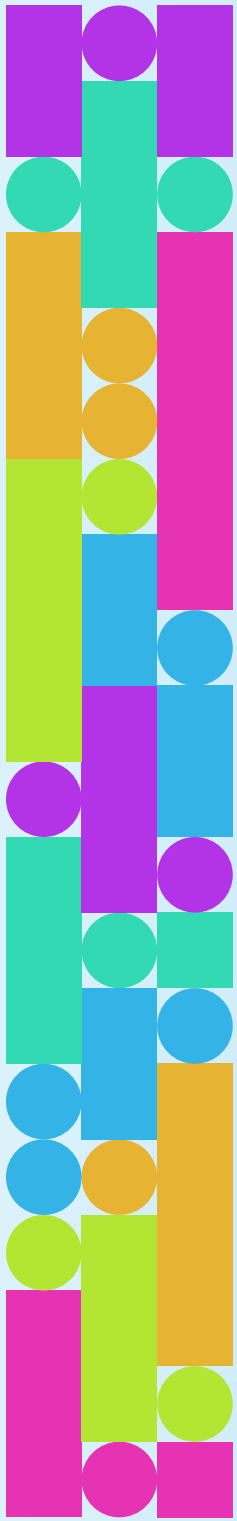
You can't teach teaching.



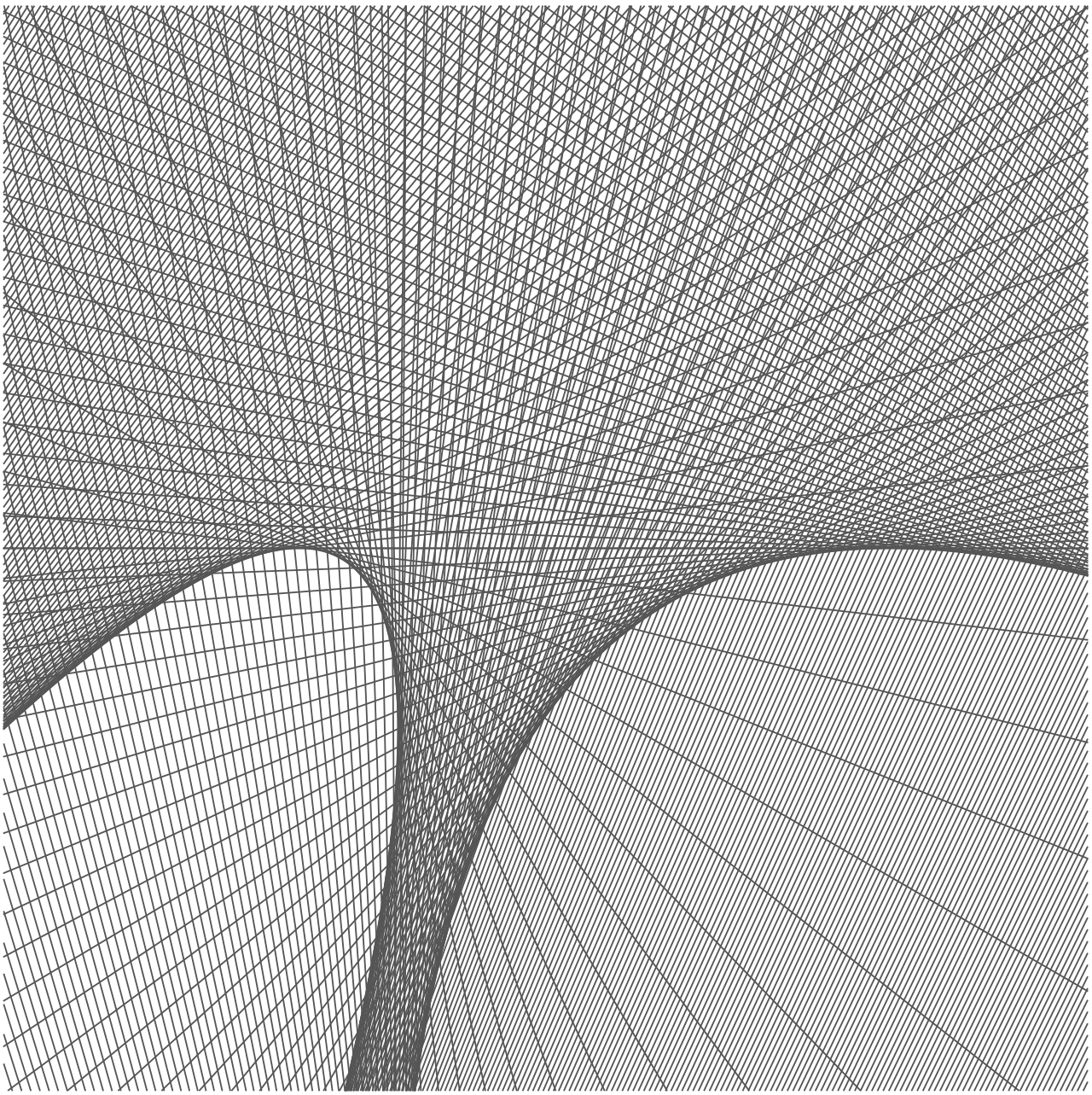
Teaching is a messy human relationship; it does not require a method. Or rather I should say, if you need a method you're probably not a very good teacher. If you don't have enough of a feeling for your subject to be able to talk about it in your own voice, in a natural and spontaneous way, how well could you understand it?

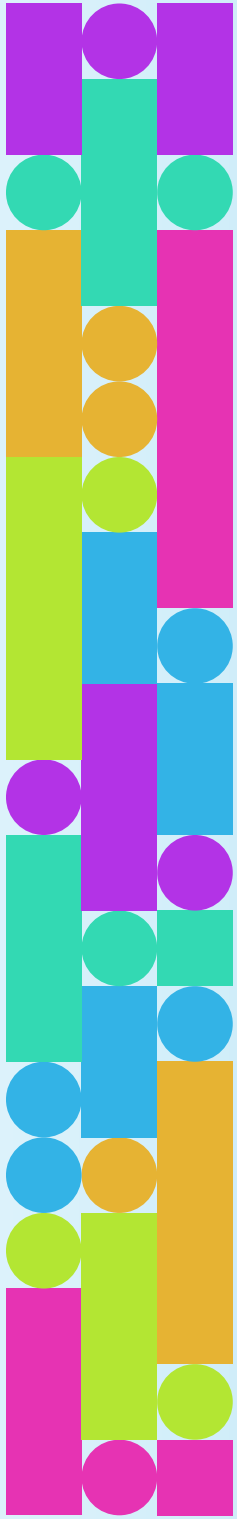




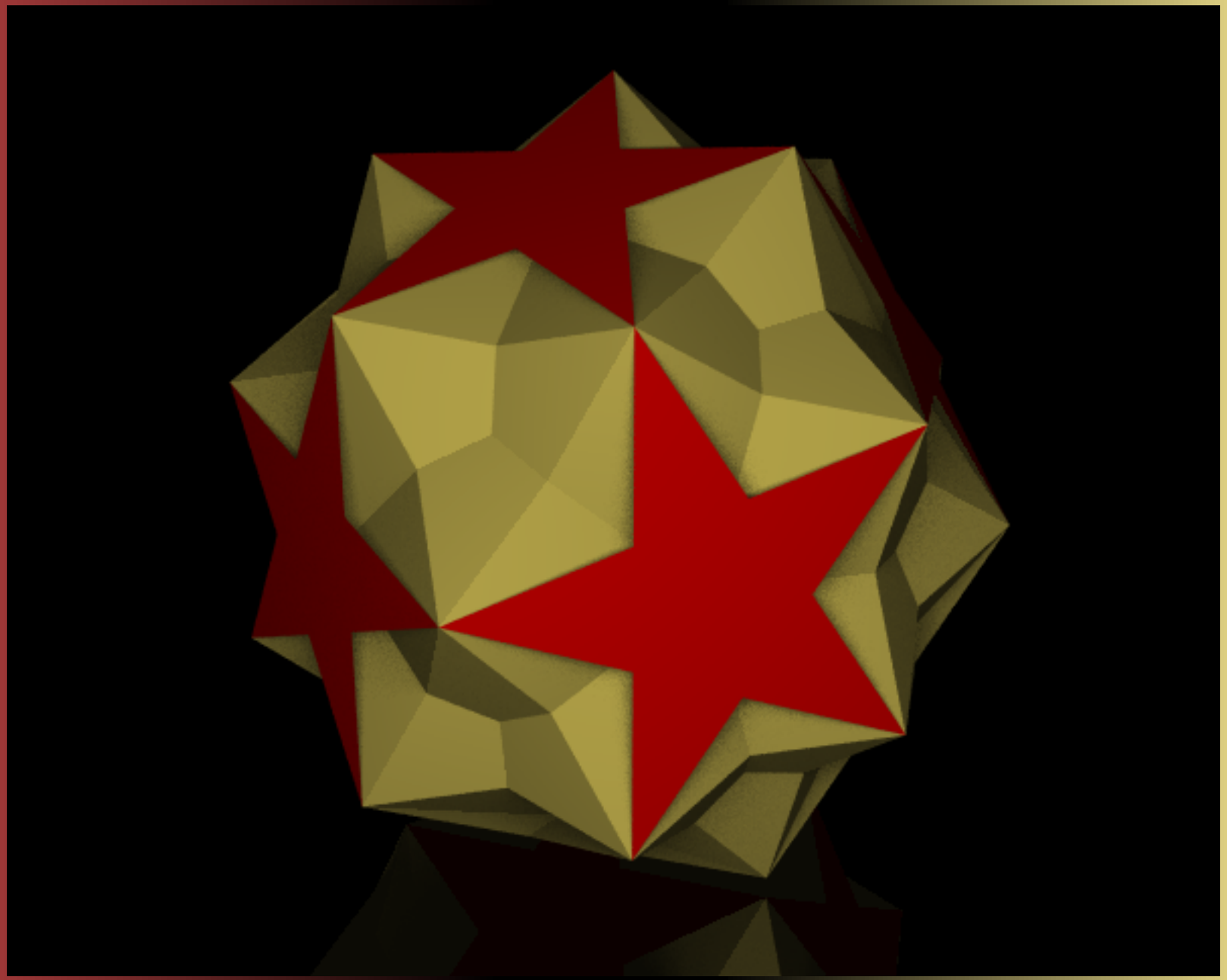


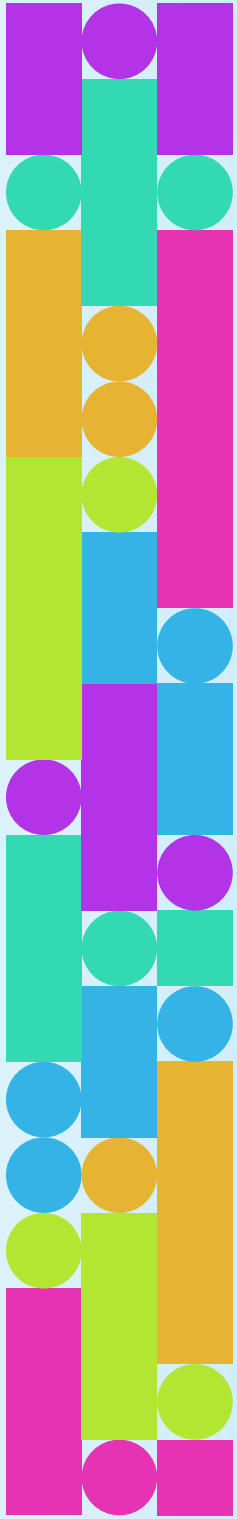
And I haven't even mentioned the lack of mathematical criticism in school. At no time are students let in on the secret that mathematics, like any literature, is created by human beings for their own amusement; that works of mathematics are subject to critical appraisal; that one can have and develop mathematical *taste*.



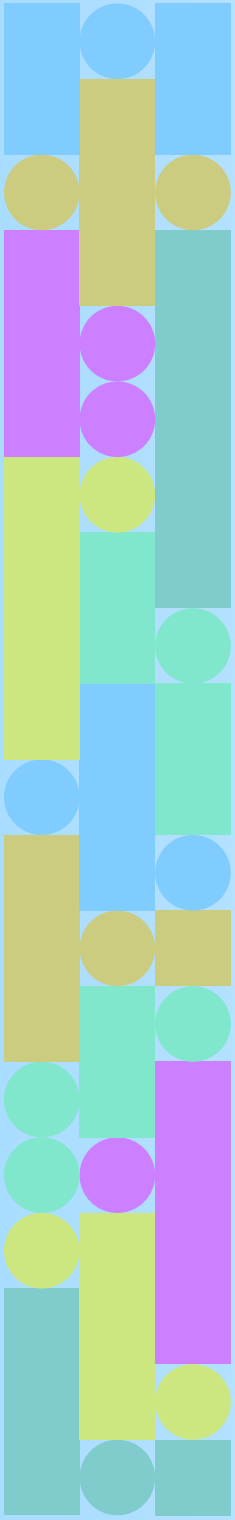


How many students taking literature classes will one day be writers? That is not why we teach literature, nor why students take it. We teach to enlighten everyone, not to train only the future professionals. In any case, the most valuable skill for a scientist or engineer is being able to think creatively and independently. The last thing anyone needs is to be *trained*.





In fact, if I had to design a mechanism for the express purpose of *destroying* a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done – I simply wouldn't have the imagination to come up with the kind of senseless, soul-crushing ideas that constitute contemporary mathematics education.

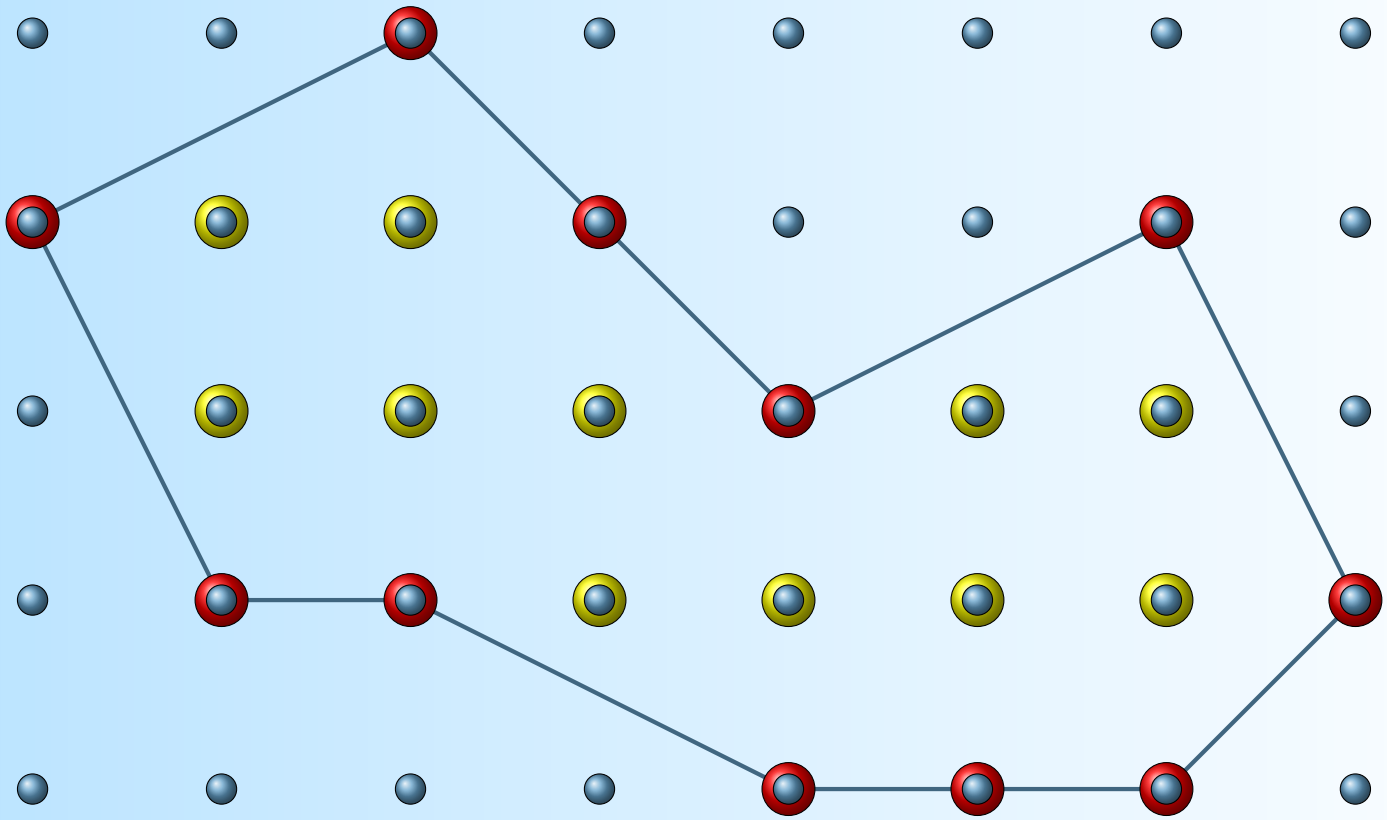


Pick's Theorem

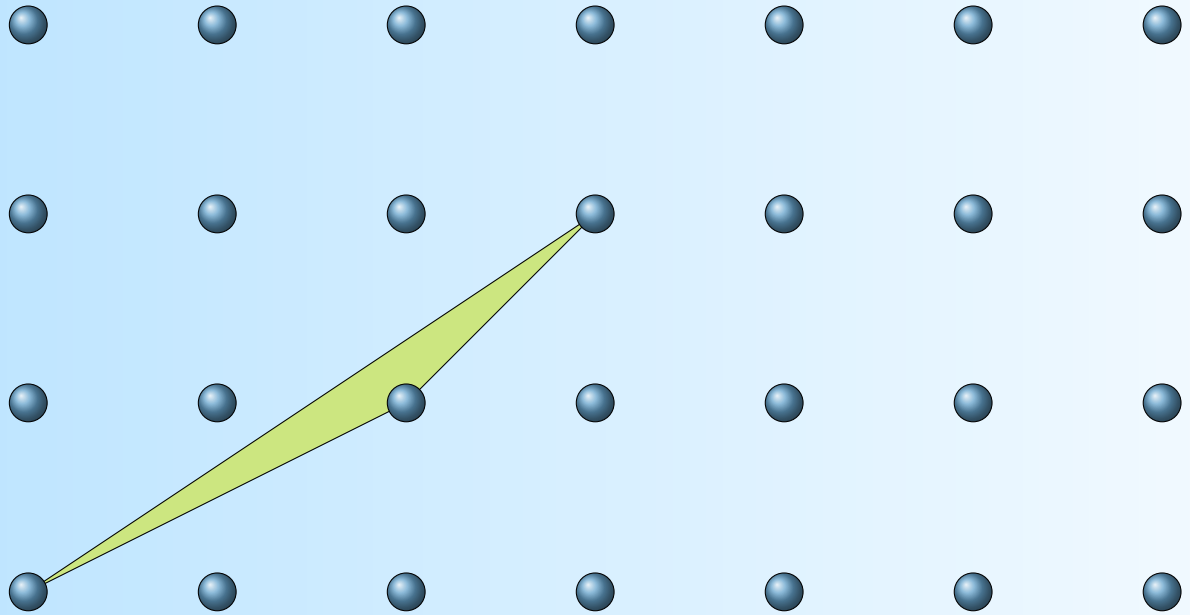
from
Proofs from THE BOOK

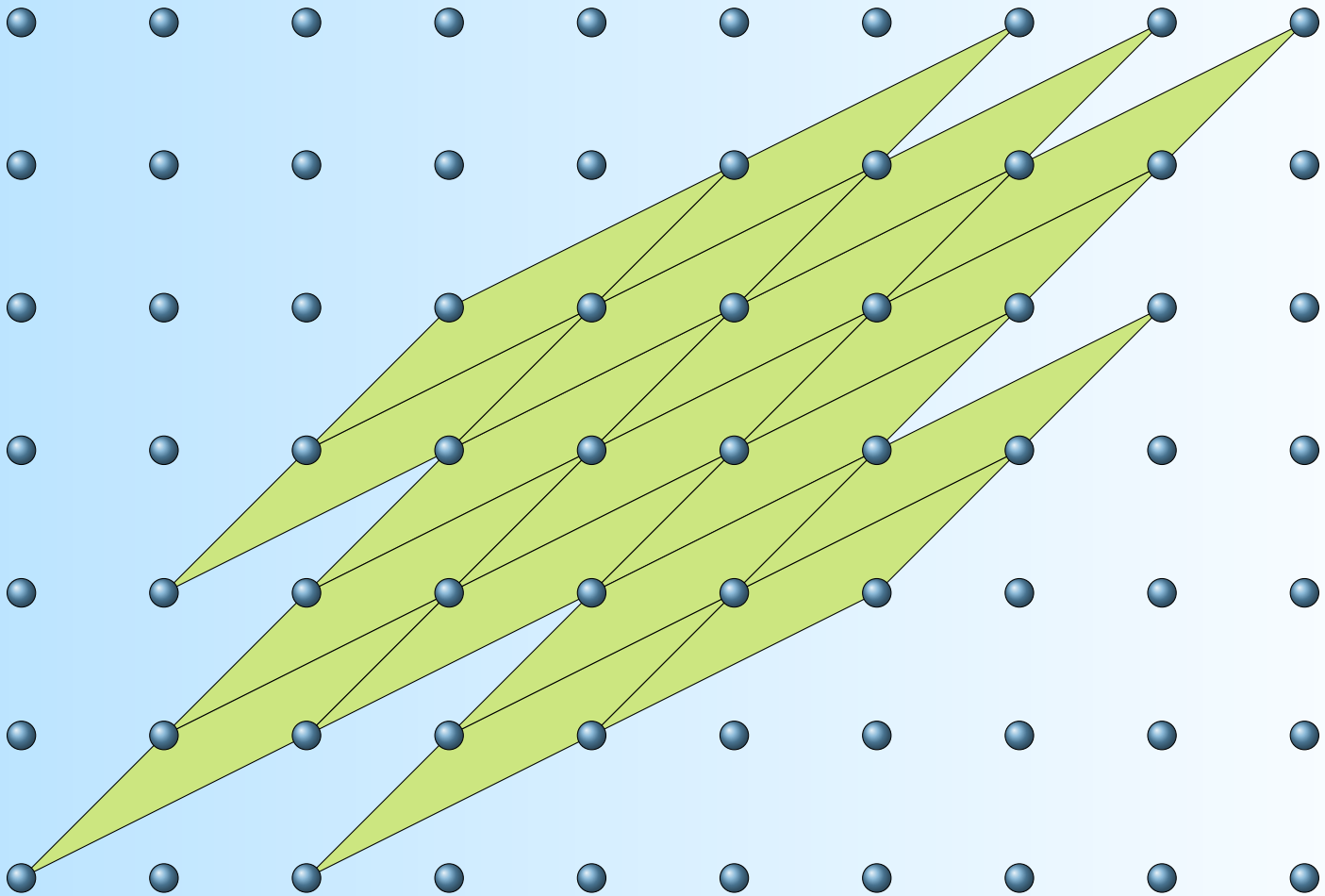
Martin Aigner
Günter M. Ziegler

$$A = V_{int} + \frac{1}{2}V_{bd} - 1$$



$$A = 11 + \frac{1}{2} \cdot 11 - 1 = \frac{31}{2}$$



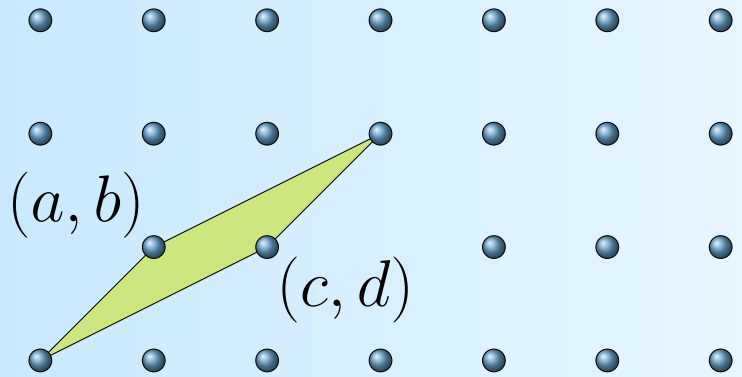


Assume $ad - bc > 0$ (otherwise, relabel). Suppose

$$(e, f) = q(a, b) + r(c, d).$$

Then

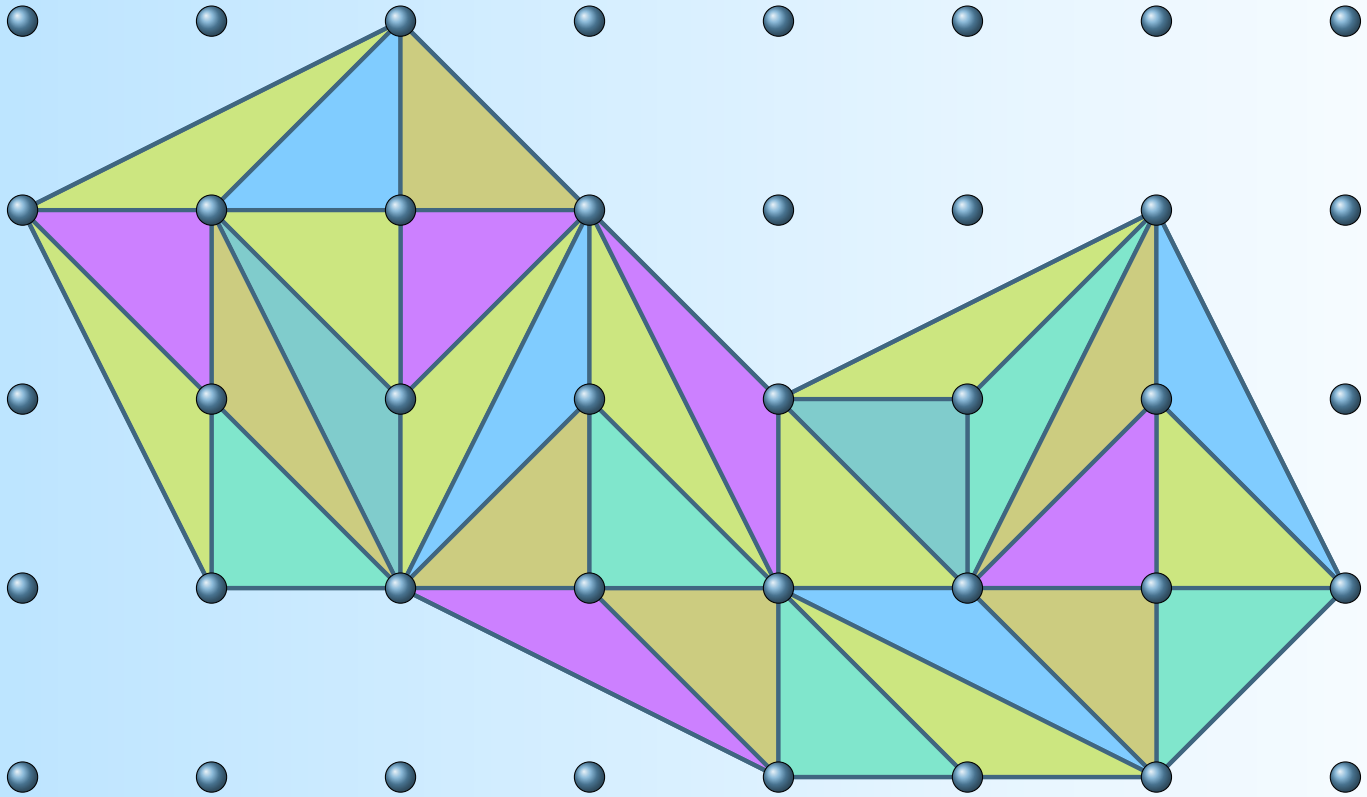
$$q = \frac{ed - fc}{ad - bc}.$$

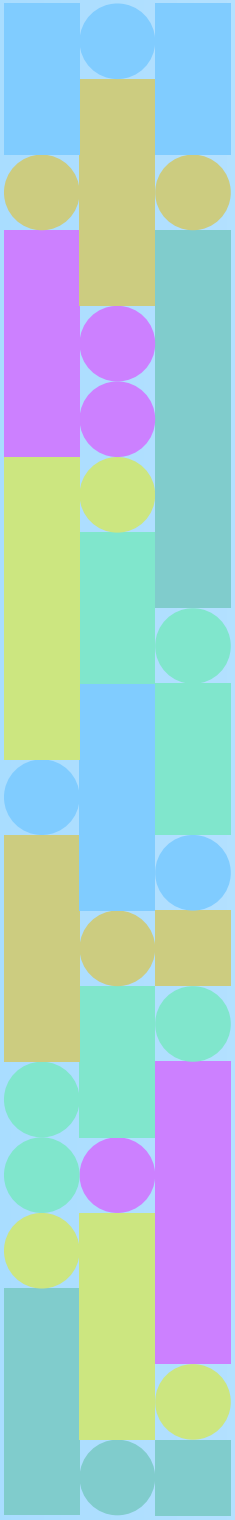


Now c and d must be relatively prime (or there would be a lattice point on the boundary). So we may find e, f with

$$ed - fc = 1.$$

But q must be an integer for *all* choices of e, f. Thus, it must be the case that $ad - bc = 1$.





Some notation:

V_{int}: number of interior vertices

V_{bd}: number of vertices on the boundary

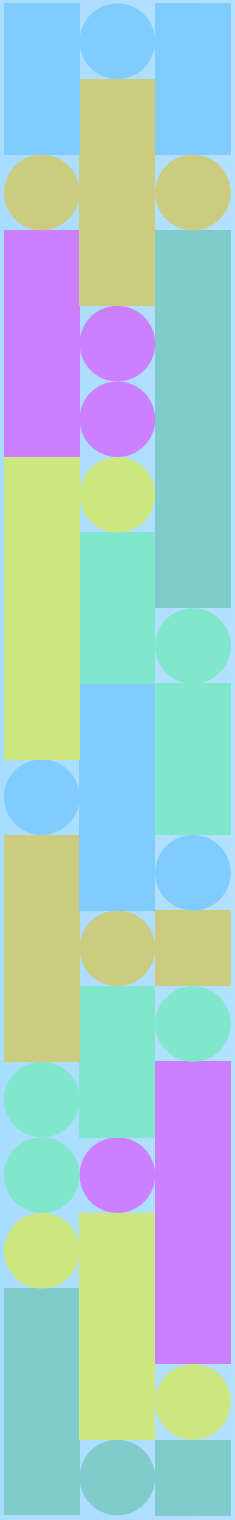
E_{int}: number of edges in the interior

E_{bd}: number of edges on the boundary

F: number of triangles

With these notations, Euler's formula for the plane becomes

$$V_{int} + V_{bd} - E_{int} - E_{bd} + F = 1.$$



Begin with Euler's formula for the plane:

$$V_{int} + V_{bd} - E_{int} - E_{bd} + F = 1.$$

Now $V_{bd} = E_{bd}$, and so

$$V_{int} - E_{int} + F = 1.$$

But

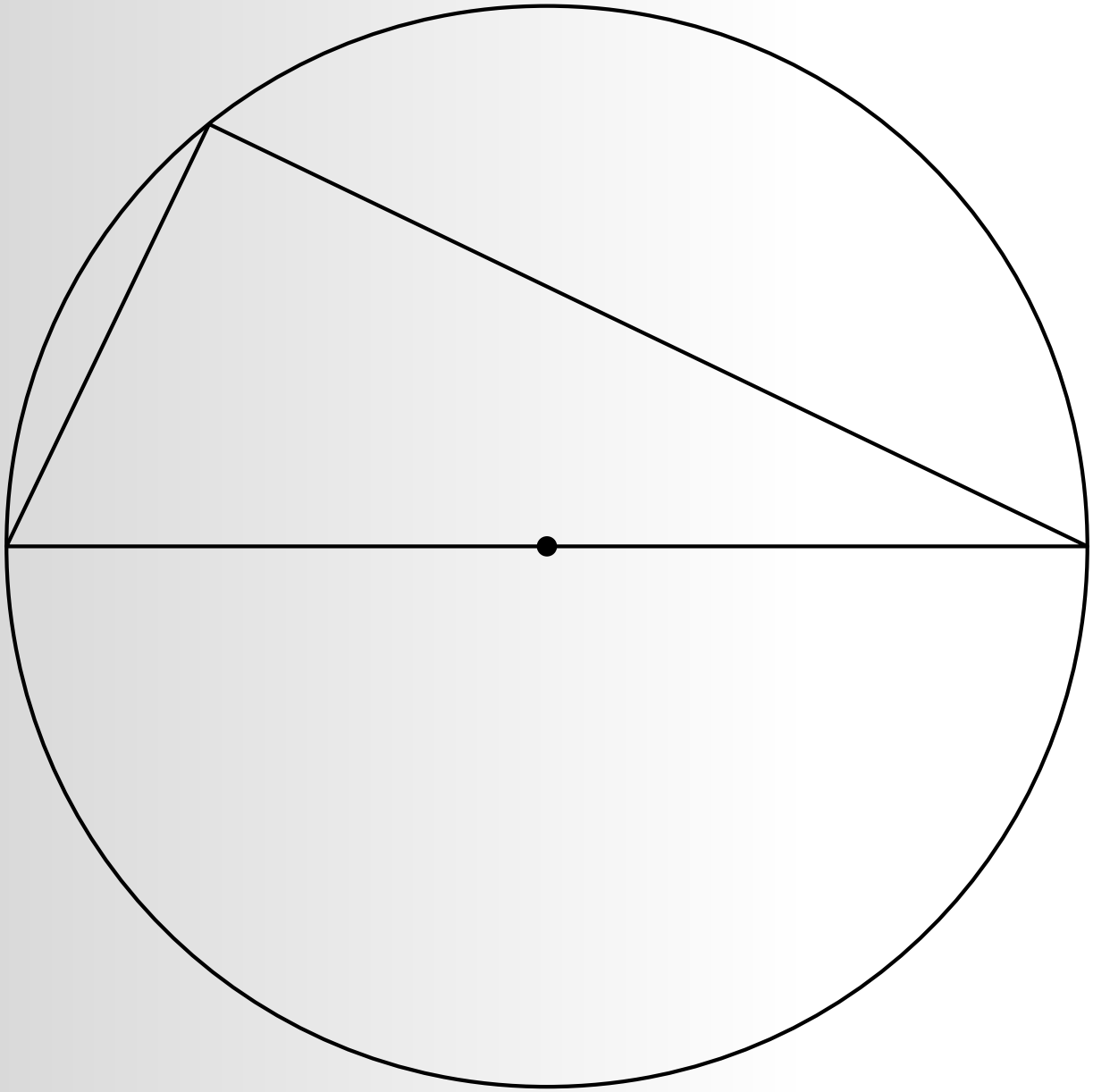
$$3F = 2E_{int} + E_{bd} = 2E_{int} + V_{bd}.$$

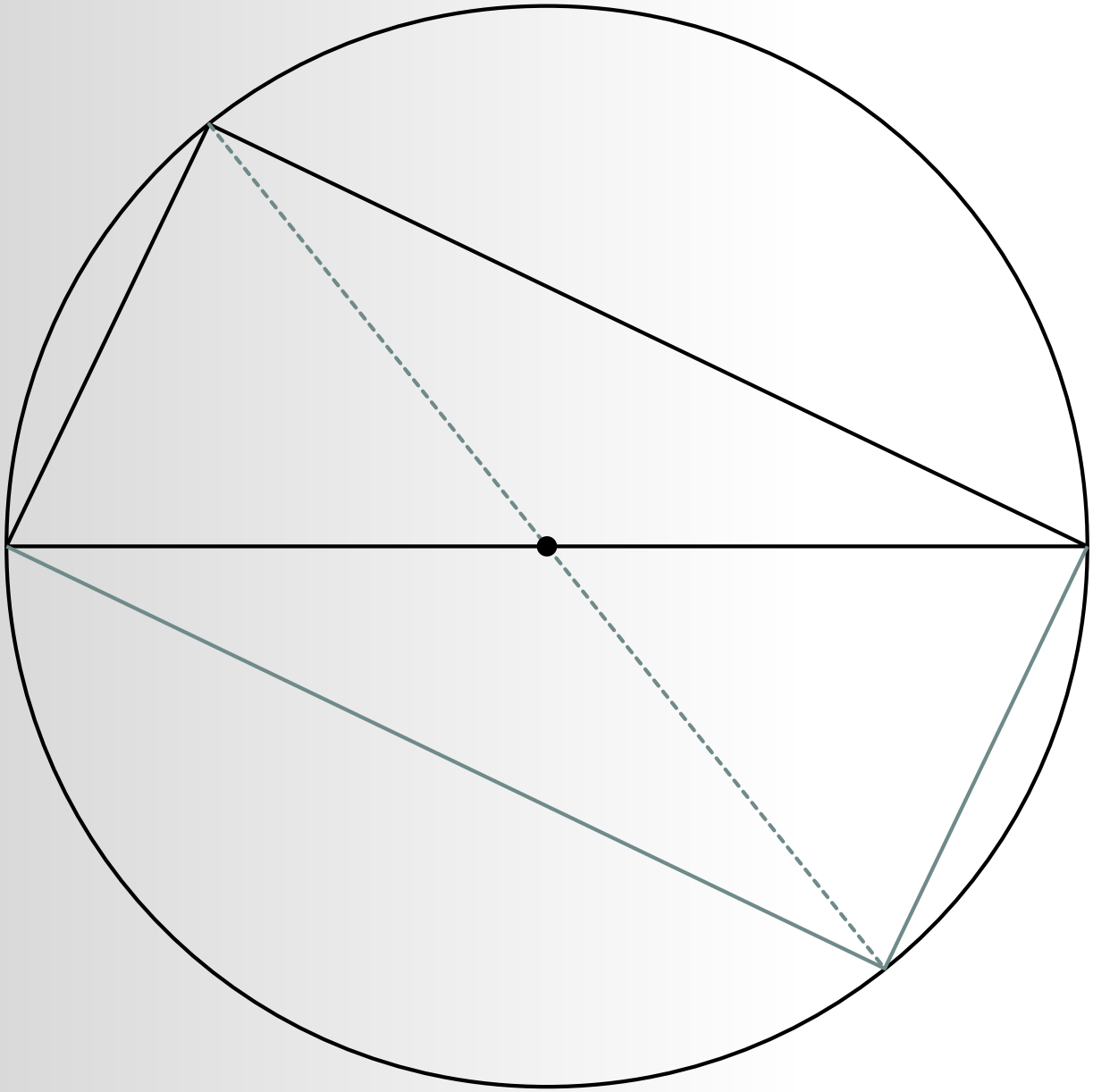
Solve for E_{int} and substitute:

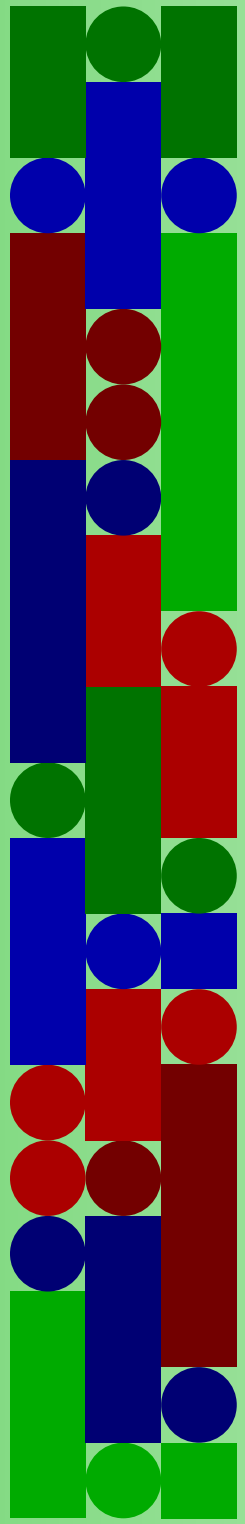
$$V_{int} + \frac{1}{2}V_{bd} - \frac{1}{2}F = 1.$$

But $F/2 = A$, and so

$$A = V_{int} + \frac{1}{2}V_{bd} - 1.$$







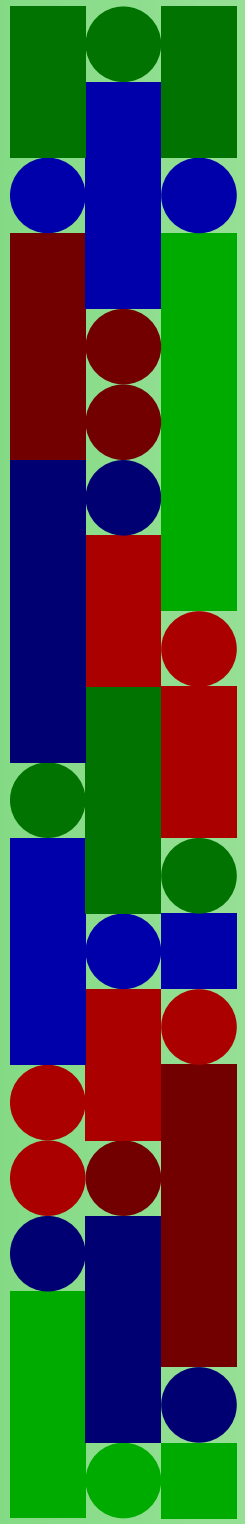
Writing Original Problems

1. Motivation: Why did you choose this problem? Where did your inspiration come from?

2. Problem Statement: Be clear! Be creative! Give it to someone else to proofread.

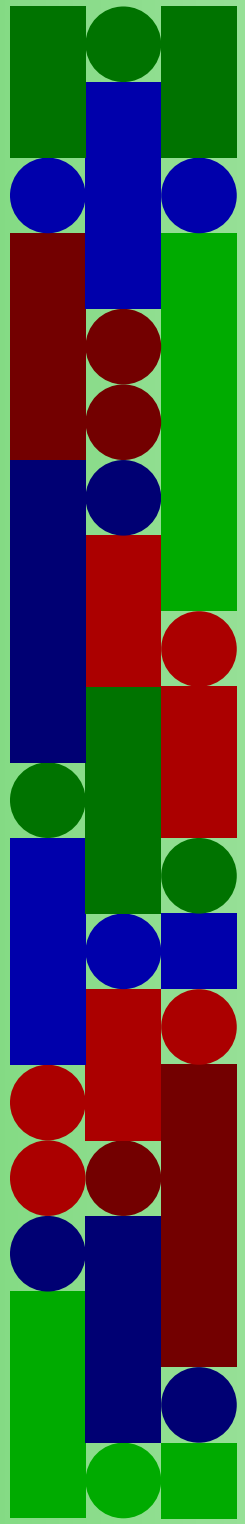
3. Solution: Be correct! Be clear! Say enough, but don't be verbose.

4. Reflection: What did you learn about yourself as a problem writer?



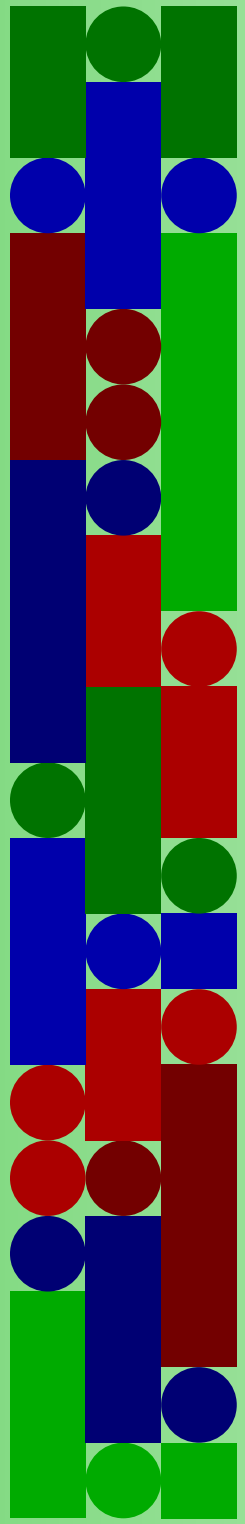
Student EL

Anyone can write tedious, difficult problems that review core math subjects, but to write problems in a novel, challenging, and refreshing manner, one must be imaginative. I feel that this creative side of math is an often overlooked aspect of the field as many believe math to be an extremely black-and-white, rigid, and boring subject.



Student BJ

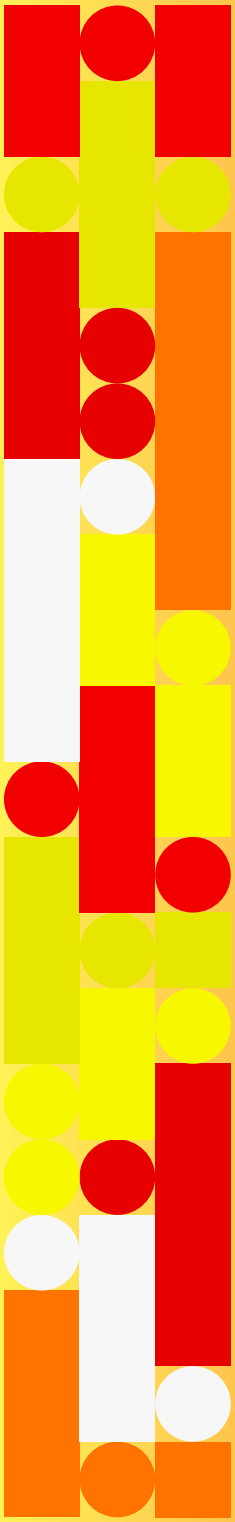
Over the course of the semester, I have come to realize the extreme degree of insight and artistic level of creativity needed to write truly good and beautiful problems. The burden of problem writing and solving truly lies more squarely on the shoulder of the writer.



For more information....

`www.vincematsko.com > talks > riga 2010`

`www.vincematsko.com > talks > korea 2012`



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\usepackage{tikz,amsmath}

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%%% Stuff in a 27.98cm by 21.57cm picture.

\end{tikzpicture}\eject

\end{document}
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