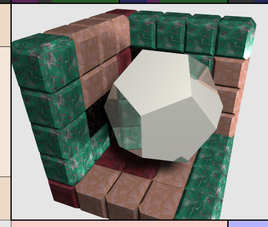
Dodecahedron Day 2012

December 5, 2012



Dodecahedron Day was founded on the premise that the teaching and learning of mathematics can be fun, engaging, and rewarding. The tradition began in 2004 when Vince Matsko and Todd Klauser took dozens of high school geometry students to a local middle school, where they introduced hundreds of younger students to the joy of constructing polyhedra. It is time to rekindle a love of mathematics in students of all ages, all around the world. So what are you waiting for? Start building!

The Platonic Solids

Platonic solids are convex polyhedra such that all faces are the same regular polygon, and the same number of polygons meet at each vertex. The first written account of the Platonic solids was Plato's dialogue *Timaeus* ($\sim$ 400 B.C.), although most were discovered earlier. There are five Platonic solids, shown at the right.

Euler's formula is valid for the Platonic solids:

$$V - E + F = 2.$$

That is, for each polyhedron, the sum of the number of vertices and the number of faces is two more than the number of edges.

In ancient Greece, they represented the five elements of earth, air, water, fire, and the heavens. Euclid discussed the Platonic solids in Book XIII of the *Elements* ($\sim$ 400 B.C.). The Platonic solids were significant in Kepler's discussion of the solar system ($\sim$ 1572). Currently, they are important in mathematics and science: fluorite crystals are in the shape of octahedra.

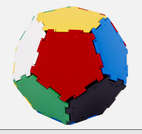
Cube

Earth

Icosahedron

Water

Dodecahedron



The Heavens

Tetrahedron

Fire

Octahedron

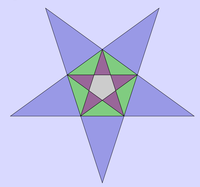
Air

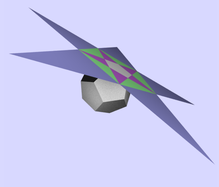
Stellations II

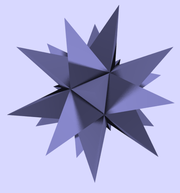
The stellations of the dodecahedron were discovered by Kepler ($\sim$1630) and independently by Poinsot ($\sim$1859). They are three of the *Kepler-Poinsot* polyhedra.

Stellations III

The last stellation of the dodecahedron is called the *great stellated dodecahedron.* Extend the sides of the pentagons of the great dodecahedron until twelve larger pentagrams are formed.

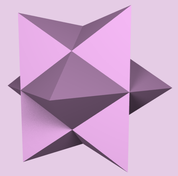






Rhombic Dodecahedron

Not all dodecahedra have pentagonal faces. The *rhombic dodecahedron* has twelve rhombic faces, each with angles of about 70.5° and 109.5°. The first stellation of the rhombic dodecahedron has the remarkable property of being a *space-filling* polyhedron; that is, space can be packed with this polyhedron with no gaps, just like cubes!



Pentominoes

Problem solving with pentominoes is also a great way to celebrate Dodecahedron Day, because there are exactly twelve shapes which can be made of five connected squares. Pentominoes were popularized by Solomon Golomb in his book *Polyominoes.* They were also discussed by Martin Gardner.

By using a 2×2 square in addition to the pentominoes, an 8×8 square may be formed. It is possible to solve this puzzle with the 2×2 piece located *anywhere* in the 8×8 square.

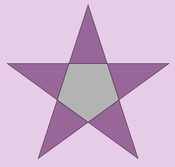
There are 65 solutions to the puzzle with the square in the center, not counting rotations and reflections. The other 64 are shown around the border of this poster.

Stellations IV

Not all dodecahedra have regular faces. Below are two examples of irregular pentagonal dodecahedra, with one stellation of each dodecahedron.

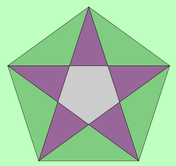
Stellations I

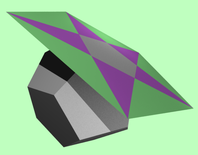
Stella is the Greek word for star. *Stellating* a polyhedron often creates a star-like polyhedron. Let's begin by stellating the dodecahedron. The gray pentagon is one of its faces.

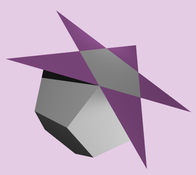


Now imagine extending the sides of the pentagon until they form a pentagram. Perform this extension on all faces of the dodecahedron. The resulting polyhedron is the *small stellated dodecahedron.*

The next stellation of the dodecahedron is called the *great dodecahedron.* Continue extending the pentagrams of the small stellated dodecahedron until twelve larger pentagons are formed.

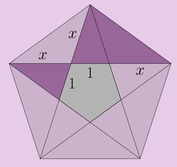






Golden Ratio

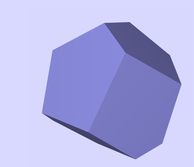
No discussion of the dodecahedron would be complete without mentioning the golden ratio, an important number in early Greek mathematics and architecture.



Recall that the interior angles of a regular pentagon have measure 108°. Using this fact, convince yourself that the shaded triangles in the figure to the left are isosceles, with base angles 72° and apex angle 36°.

Since the two shaded triangles are similar, we may form the ratios $\frac{x}{1} = \frac{1+x}{x}$. This may be rewritten as $x^2 = x + 1$. The positive root of this quadratic equation, $\frac{1+\sqrt{5}}{2}$, is often symbolized by ϕ and is called the *golden ratio.* Whenever regular pentagons are involved, ϕ is sure to be nearby.

Kepler called this polyhedron the *echinus minor icosaëdricus*, meaning the "smaller twenty-faced hedgehog." Modern names for the stellations of the dodecahedron were introduced by Cayley in 1859.

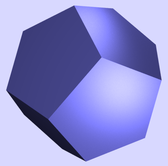


Register your participation

Our goal is to have 1,000,000 students and teachers all around the world participate in Dodecahedron Day 2012 activities. Be a part of ramping up to this global event by participating in Dodecahedron Day 2012! Visit coolhub.imsa.edu to register your participation.

Dodecahedron Day at IMSA

Dodecahedron Day 2012 is a project led by Dr. Vince Matsko, mathematics faculty member at IMSA, one of the world's premier institutions for talented mathematics and science students. Visit www.imsa.edu for more information.



Special Thanks...

...to Lucinda Dittmer, Rob Flemming, Juli Geldner, Jim Gerry, Dr. Max McGee, Britta McKenna, Cathy Veal, and all those teachers and students who participated in Dodecahedron Day 2011!

References

Historical references are discussed in Peter Cromwell's *Polyhedra,* Cambridge University Press, 1997 (ISBN 0 521 55432 2). This poster was produced at the Illinois Mathematics and Science Academy using Till Tantau's TikZ package in LaTeX. Three-dimensional graphics were created using POV-Ray (www.povray.org). The pdf file for this poster may be found at www.dodecahedronday.org. This poster may be printed free of charge for educational purposes. Under no circumstances may it be sold or otherwise used for monetary gain.