The Archimedean Solids

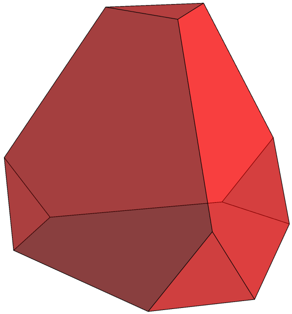
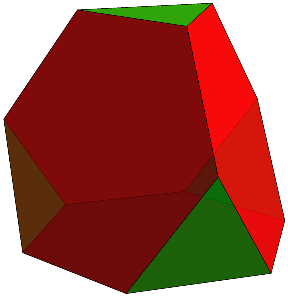
Archimedean Solids

An Archimedean solid:

1. Has faces which are regular polygons,
2. Has the same arrangement of faces at each vertex, and
3. Possesses maximal symmetry.

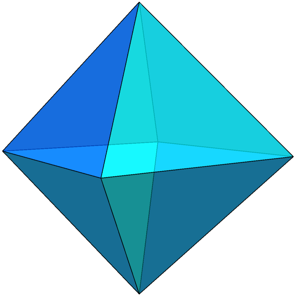
We will not discuss the last requirement in detail, but will simply enumerate the Archimedan solids without proof.

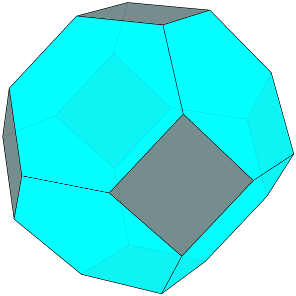
The Truncated Tetrahedron

We may truncate a tetrahedron until all faces are regular polygons.

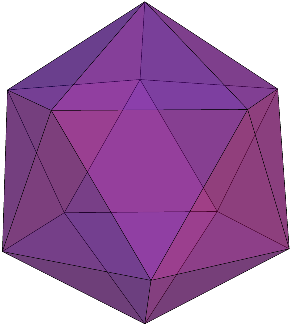
The Truncated Octahedron

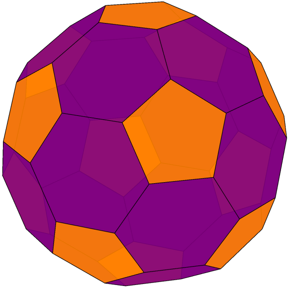




We may also truncate the octahedron until the triangles become regular hexagons.

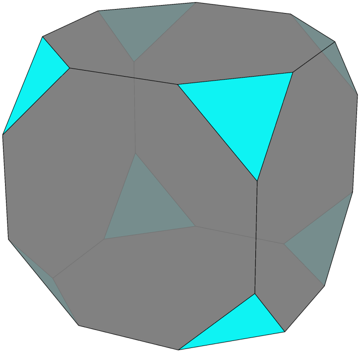
The Truncated Icosahedron



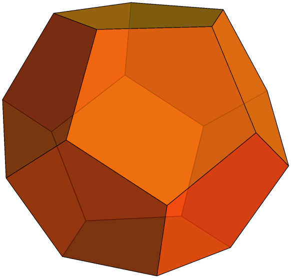


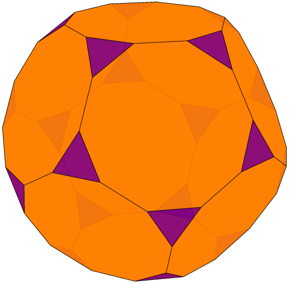
Similarly, we may truncate the icosahedron until the triangles become regular hexagons.

The Truncated Cube



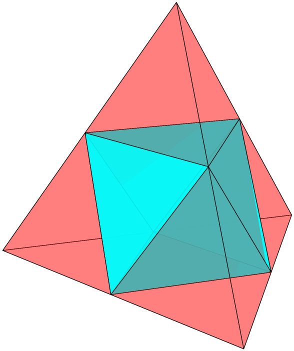
The Truncated Dodecahedron





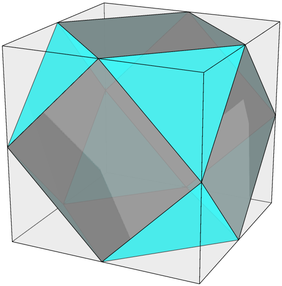
We may truncate the dodecahedron until the pentagons become regular decagons.

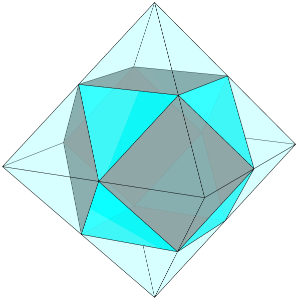
Complete Truncation



The tetrahedron may be completely truncated to produce an octahedron.

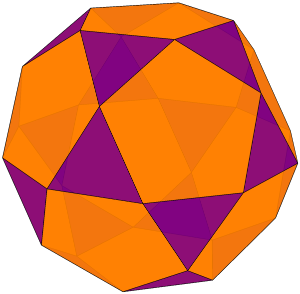
The Cuboctahedron





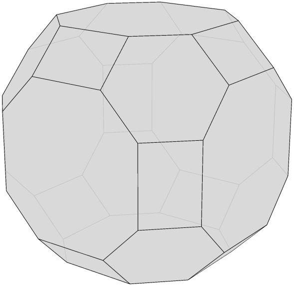
The cube (or octahedron) may be completely truncated to produce a cuboctahedron.

The Icosidodecahedron



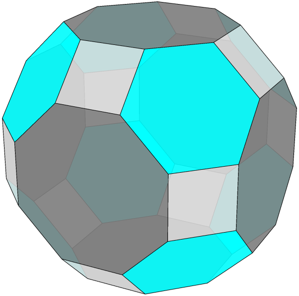
The icosahedron (or dodecahedron) may be completely truncated to produce an icosidodecahedron.

Truncating a Cuboctahedron



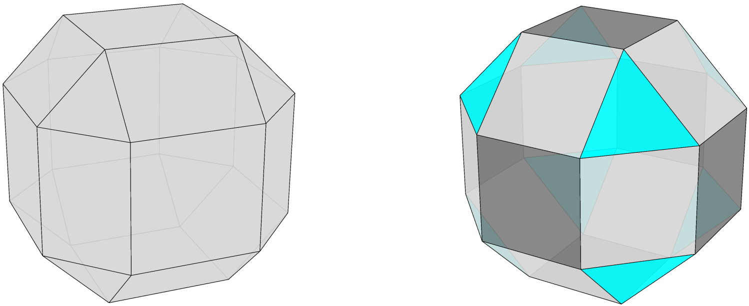
Partially truncating a cuboctahedron does not produce a polyhedron with regular faces.

The Rhombitruncated Cuboctahedron



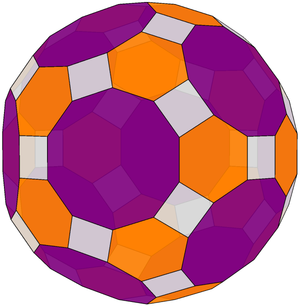
However, making slight adjustments produces the rhombitruncated cuboctahedron.

The Rhombicuboctahedron



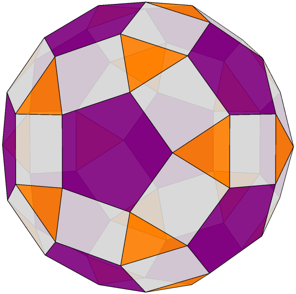
The rhombicuboctahedron is produced by adjusting a completely truncated cuboctahedron.

The Rhombitruncated Icosidodecahedron



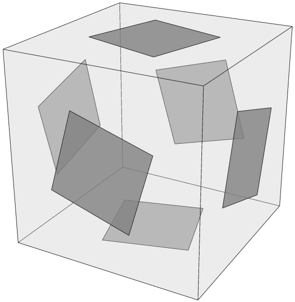
Adjusting a partially truncated icosidodecahedron results in a rhombitruncated icosidodecahedron.

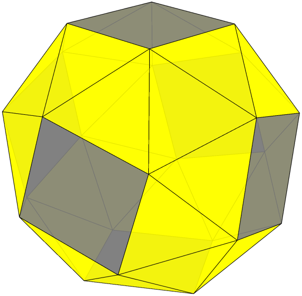
The Rhombicosidodecahedron



Adjusting a completely truncated icosidodecahedron creates a rhombicosidodecahedron.

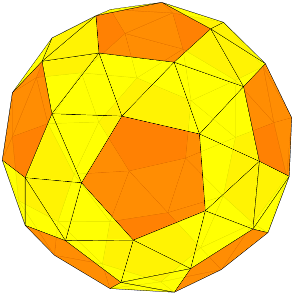
The Snub Cube

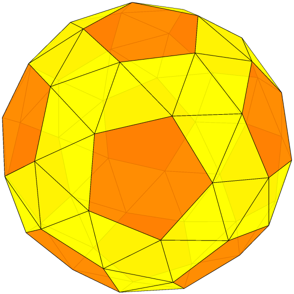




A snub cube is created by slight rotating squares on the faces of a cube.

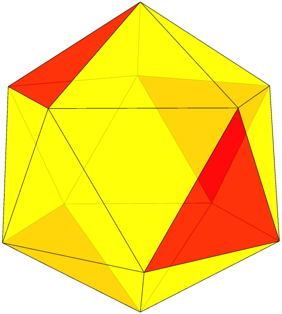
The Snub Dodecahedron

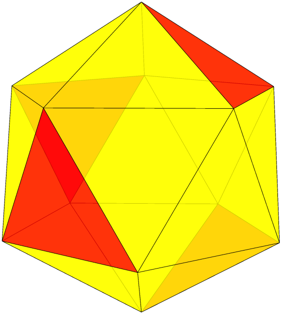




A snub dodecahedron is similarly created. Snub polyhedra occur in enantiomorphic pairs.

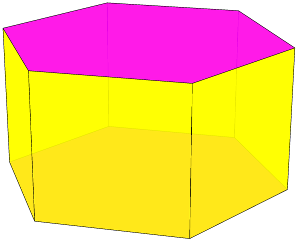
Another Look at the Icosahedron





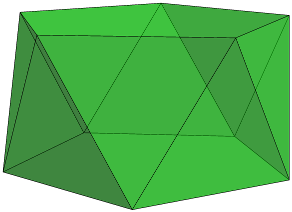
A snub tetrahedron is actually an icosahedron!

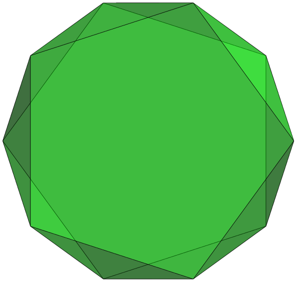
The Prisms



There are two infinite families of Archimedean solids. One family is the prisms.

The Antiprisms

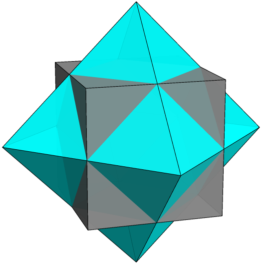




The other infinite family is the antiprisms.

Duality

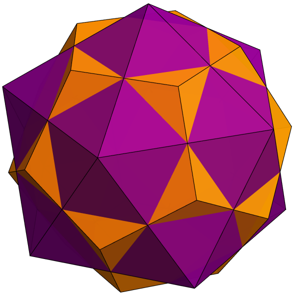
The Cube and Octahedron



That the cube and octahedron are *dual* means:

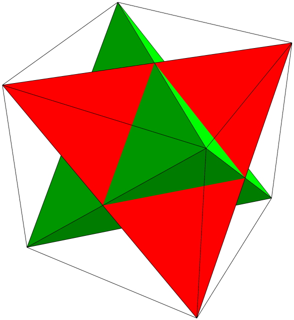
1. The number of faces on the cube is the same as the number of vertices on the octahedron (and *vice versa*);
2. Each edge of the octahedron is perpendicular to an edge of the cube.

The Dodecahedron and Icosahedron



The dodecahedron and icosahedron are also dual to each other.

The Tetrahedron



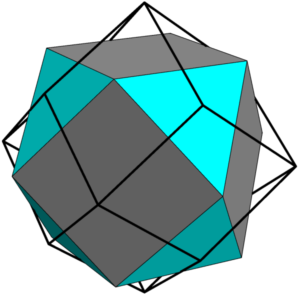
The tetrahedron is said to be *self-dual.* The compound of the tetrahedron and its dual is called the *stella octangula.* It is a stellation of the octahedron.

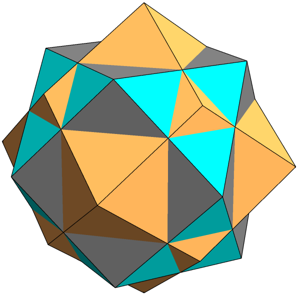
Duality and the Platonic Solids

Note that the dual of a Platonic solid is also a Platonic solid. The duals of Archimedean solids are more complex.

Polyhedron	Dual
Tetrahedron	Tetrahedron
Cube	Octahedron
Octahedron	Cube
Dodecahedron	Icosahedron
Icosahedron	Dodecahedron

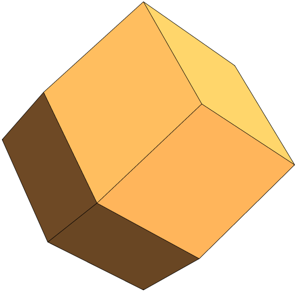
Dual of the Cuboctahedron





To find the dual of the cuboctahedron, we perpendicularly bisect its edges.

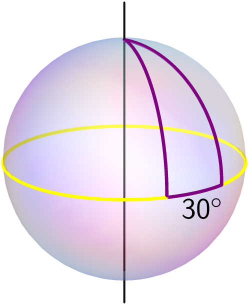
The Rhombic Dodecahedron



The dual of the cuboctahedron is the *rhombic dodecahedron*. It has 12 faces because the cuboctahedron has 12 vertices.

Geometry on a Sphere

Points and Lines



We say that a *line* is a great circle on the sphere. A *great circle* is a circle whose center coincides with the center of the sphere. A *line segment* is an arc of a line.

Non-Euclidean Geometry

Geometry on a sphere is an example of a *non-Euclidean geometry* – a system of geometry where the usual axioms and theorems of Euclidean geometry do not apply. For example:

Observation 1: *Two points do not uniquely determine a line.* Infinitely many lines pass through opposite points on a sphere. If two points are not opposite, then there is exactly one line passing through both points.

Parallel Lines

Observation 2: *There are no parallel lines.* Here, we use the definition of parallel lines as two lines which do not intersect.

Note that *any* two distinct lines (that is, great circles) on a sphere intersect in exactly two points. As a result, *no* two lines on a sphere are parallel.

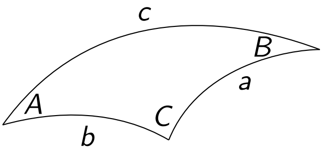
Measuring Line Segments

Since line segments are arcs of great circles on a sphere, it makes sense to measure line segments in either degrees or radians. In other words, much of the geometry on the sphere does not depend on how large the sphere is, or in what units distances are measured. As a result:

Observation 3: *The six parts of a triangle are all angles.* Thus, theorems such as the Pythagorean theorem do not apply, and we must rethink our ideas about similarity and congruence.

And because all six parts of a triangle are angles, trigonometry is crucial to any study of spherical triangles.

Area of a Spherical Triangle



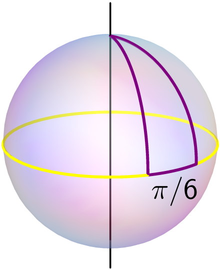
If the radius of the sphere is ρ, the area of spherical ΔABC is

$$\rho^2(A + B + C - \pi).$$

Observation 4: *In a spherical triangle,*

$$A + B + C > \pi.$$

An Example

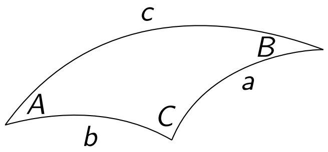


The area is

$$\rho^2\left(\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{6}-\pi\right)=\frac{\pi}{6}\rho^2=\frac{1}{24}4\pi\rho^2,$$

or $1/24$ the area of the sphere, as expected.

Spherical Trigonometry



$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Similarity and Congruence

Because of a cosine law,

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c,$$

we see that the angles of a spherical triangle determine the sides. Thus,

Observation 5: *Similarity and congruence are the same concept.* Unlike Euclidean geometry, where knowing the angles in a triangle only determines its shape, the size is *also* determined in spherical geometry.

Non-Euclidean Geometry: Summary

Observation 1: Two points do not uniquely determine a line.

Observation 2: There are no parallel lines.

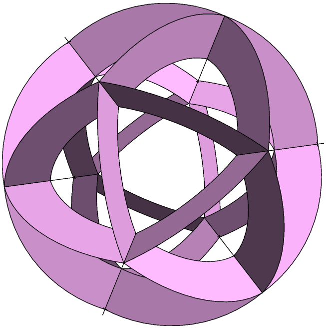
Observation 3: The six parts of a triangle are all angles.

Observation 4: In a spherical triangle,

$$A + B + C > \pi.$$

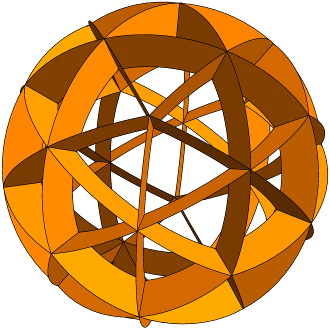
Observation 5: Similarity and congruence are the same concept.

A Spherical Icosahedron



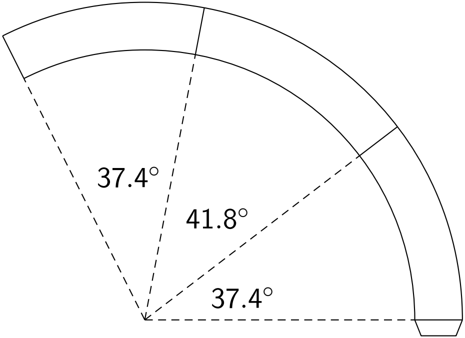
The angles in these spherical triangles are all 72°. The sides are approximately 63.4°.

A Spherical Dodecahedron



The angles in these spherical triangles are 72°, 60°, and 60°. The sides have lengths 37.4° and 41.8°, approximately.

Building a Geodesic Dodecahedron



Nets can then be created knowing the sides of the spherical triangles.