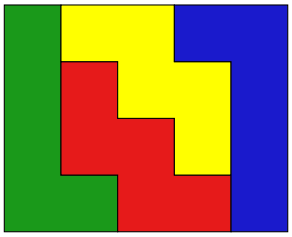
Some Tiling Problems

Note: Not all of the problems posed are solved! When asked to find the smallest rectangle able to made with given pentominoes, find the smallest you can. It is much more difficult to *prove* your rectangle is the smallest.

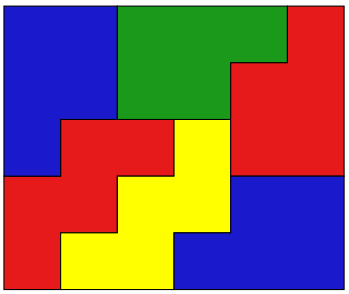
1. What is the smallest rectangle which may be tiled with the L and W pentominoes, using at least one of each?

   Solution:



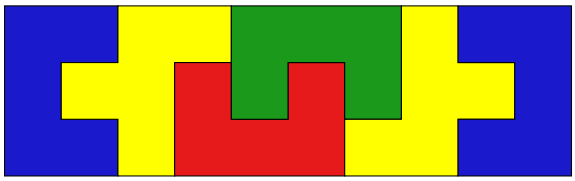
2. What is the smallest rectangle which may be tiled with the P and W pentominoes, using at least one of each?

   Solution:



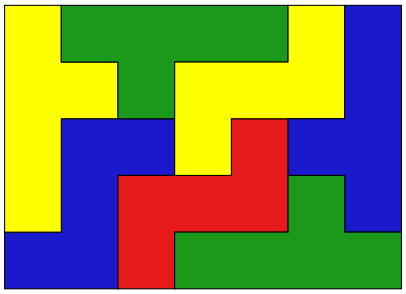
3. What is the smallest rectangle which may be tiled using the U and F pentominoes, using at least one of each?

   Solution:



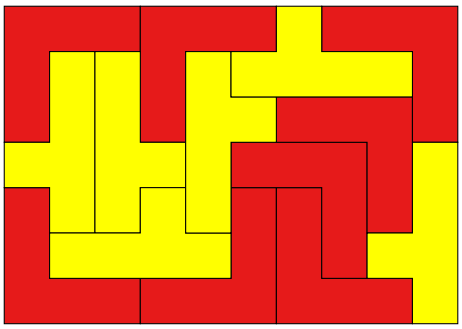
4. What is the smallest rectangle which can be made with the Y and Z pentominoes, using at least one of each?

   Solution:



5. What is the smallest rectangle which may be made with the V and Y pentominoes, using at least one of each?

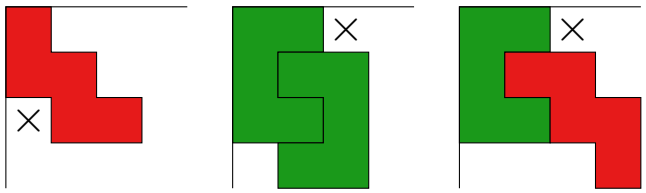
   Solution:



6. Show that it is impossible for the U and W pentominoes to tile *any* rectangle.

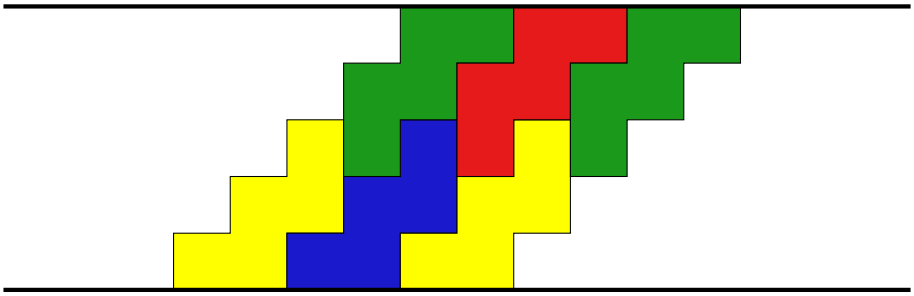
   Solution:

   There are essentially three ways to fill in a corner of a square. Each of the three ways requires a W pentomino to cover the square marked with an $\times$, which creates a similar square later on, perpetuating the difficulty.



7. Show that it is possible for the W pentomino to tile an infinite strip.

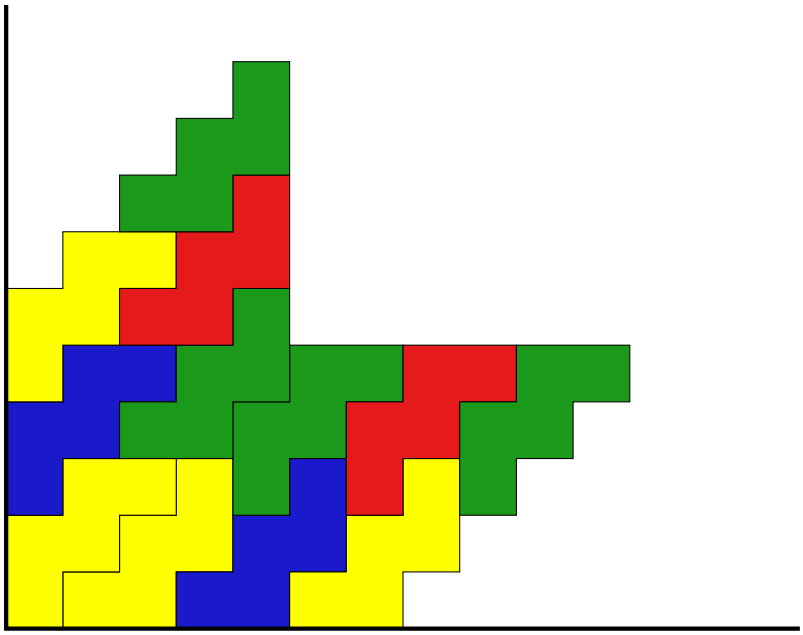
   Solution:



8. Can the W pentomino tile a quadrant?

   Solution:

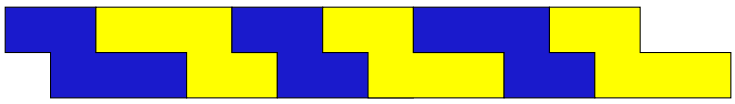
   This is easy to see once we know the way to tile an infinite strip with the W pentomino.



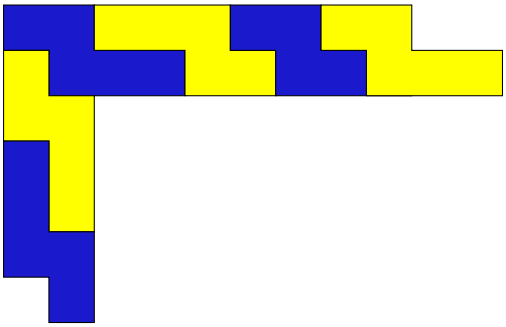
9. Show that the N pentomino can tile a quadrant. Can it tile an infinite strip?

   Solution:

   It is easy to see that the N pentomino can tile an infinite strip.



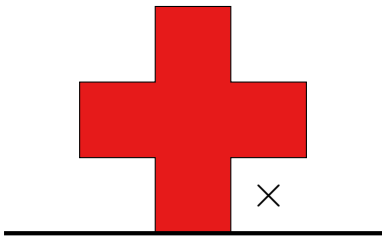
The N pentomino can also tile a quadrant, as shown below.



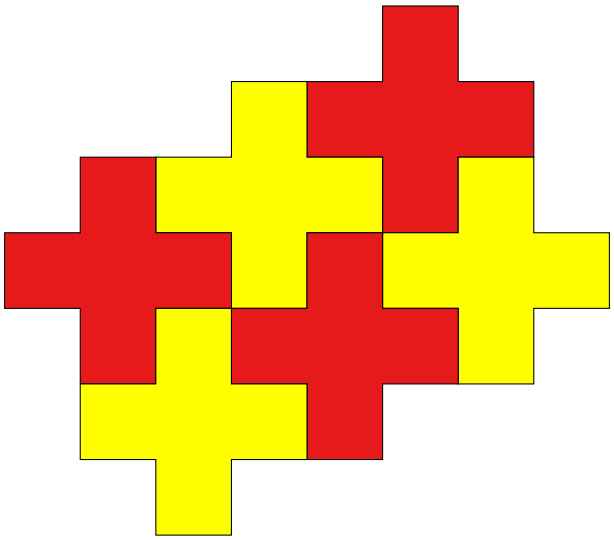
10. Show that the X pentomino cannot tile an infinite strip, but that it can tile the plane.

SOLUTION:

It is not difficult to show that the X pentomino cannot tile an infinite strip, since there is one way an edge of the X pentomino can touch the edge of a strip. In that case, the square marked with an $\times$ can not be covered.



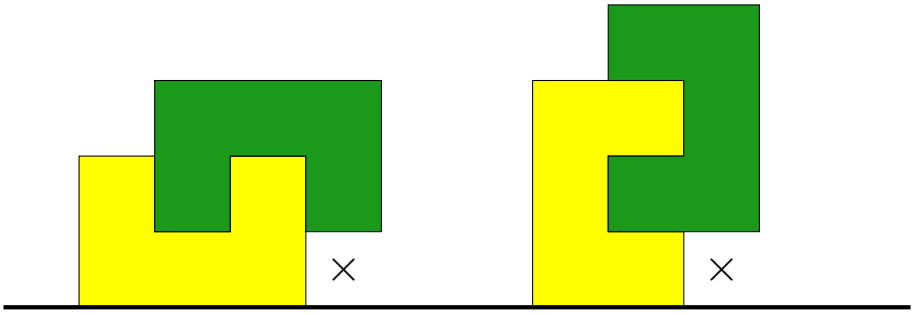
We also see that the X pentomino can tile the plane.



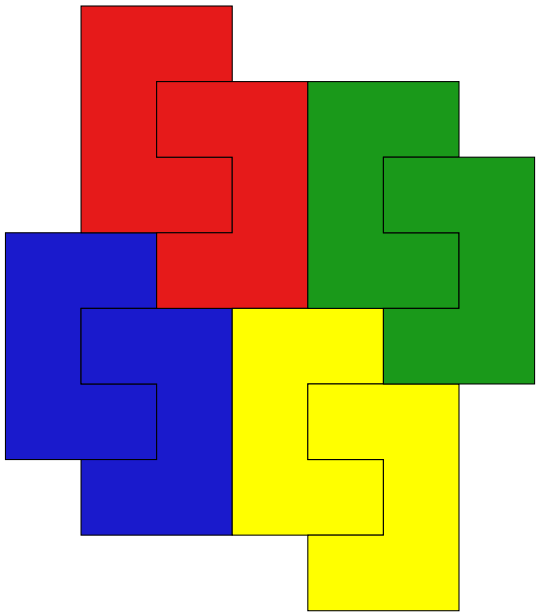
11. Show that the U pentomino cannot tile an infinite strip, but that it can tile the plane.

    SOLUTION:

    It is not difficult to show that the U pentomino cannot tile an infinite strip, since there are essentially two ways an edge of the U pentomino can touch the edge of a strip. In neither case can the square marked with an $\times$ be covered.



    To see that the U pentomino can tile the plane:



12. Create your own tiling problems!