WORKSHOP PROBLEMS

1. Complete the following table of compositions of symmetries of the triangle.

0	Ι	R_{120°	R_{240°	M_{30°	M_{90°	$M_{120^{\circ}}$
Ι	Ι	R_{120°	$R_{240^{\circ}}$	M_{30°	$M_{90^{\circ}}$	$M_{120^{\circ}}$
$R_{120^{\circ}}$	$R_{120^{\circ}}$					
$R_{240^{\circ}}$	R_{240°					
M_{30°	M_{30°					
M_{90°	$M_{90^{\circ}}$	M_{30°		R_{120°		
$M_{120^{\circ}}$	$M_{120^{\circ}}$					

2. Complete the following table of compositions of reflections of the square.

0	$M_{0^{\circ}}$	$M_{45^{\circ}}$	$M_{90^{\circ}}$	$M_{135^{\circ}}$
$M_{0^{\circ}}$				
$M_{45^{\circ}}$				
M_{90°				
$M_{135^{\circ}}$				

3. Perform the following calculations:

(a)
$$\begin{bmatrix} 3 & -5 \\ -2 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

(b) $\begin{bmatrix} 3 & -5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 \\ 3 & -4 \end{bmatrix}$

4. What geometric transformations do the following matrices describe?

(a)
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Matsko

- 5. Find matrices which describe the symmetries of the square.
- 6. Find the area of the parallelogram in terms of a, b, c, and d.



7. Verify a few of the entries in the table you made using the matrix description of the transformations and matrix multiplication. For example, $M_{0^{\circ}} \circ M_{45^{\circ}} = R_{270^{\circ}}$, and

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

- 8. What is the equation of the line obtained when y = -x+2 is transformed by $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$?
- 9. Find the affine transformation corresponding to a *clockwise* rotation of 90° about the point $\begin{pmatrix} -2\\ 1 \end{pmatrix}$.
- 10. Find the affine transformation corresponding to a reflection about the line $y = \frac{1}{2}x + 1$.
- 11. Find the matrix which describes rotating the cube 120° around the vertex (1, -1, 1).

12. Find the matrix which describes the reflection about the following plane of symmetry:



13. Find the matrix in three dimensions which represents the affine transformation

$$\mathbf{A}\begin{pmatrix} x\\ y \end{pmatrix} = \begin{bmatrix} 1 & 2\\ 3 & 4 \end{bmatrix} \begin{pmatrix} x\\ y \end{pmatrix} + \begin{pmatrix} 5\\ 6 \end{pmatrix}.$$