Conceptual Problems, Exam 1:

1. Decide, with proof, whether $f(x) = \dfrac{\sin x}{x^3}$ is even, odd, or neither.
2. The displacement curve $d(t)$ and velocity curve $v(t)$ of a particle satisfy the conditions that $d(1) = -2$, d is concave down, and $(d(t))^2 = v(t)$. Describe the path of the particle.

Conceptual Problems, Exam 2:

3. Suppose that $g'(x) \leq -2$ on $[-3, 0]$, and that $g(0) = 5$. What can be said about $g(-3)$? Your answer to this question should be written in paragraph form with complete sentences, with symbols used appropriately.
4. Create a function f defined on all real numbers with the following properties:
   - f is concave down on $(-\infty, 0)$;
   - f is concave up on $(0, \infty)$;
   - $f(x) \geq f'(x)$ for all real numbers x.

   You do not need an explicit formula; a sketch is fine. Justify your answer appropriately.
5. Create a sign chart for $f'(x) = \cos(x)\sin(2x)$. Sketch a graph of f.

Conceptual Problems: Exam 3:

6. Rigorously find the following limit: $\displaystyle\lim_{x\to-\infty} \frac{x^2-1}{x^3+4x}$.
7. Suppose that a function is defined as

   $$f(x) = \begin{cases} x^2 + ax + b, & x < 1, \\ 2bx + a, & x \geq 1. \end{cases}$$

   Find all values for a and b which make f both continuous and differentiable on $\mathbb{R}$.
8. Given that $\dfrac{d}{dx}e^x = e^x$, find $\dfrac{d}{dx}e^{-x}$.

   (Note: Students had not learned the derivative of the exponential function at this point.)

Conceptual Problems, Exam 4:

9. Find all values of a and k such that $y(x) = ax^k$ is a solution to the differential equation

$$x^6 y'' - x^5 y' = 0.$$

10. Oh no! You totally forgot the derivative of $y = \sqrt{x}$. But no matter. You know that $y = \sqrt{x}$ is the inverse function of $y = x^2$ on $[0, \infty)$. So, *only* using the fact that

$$\frac{d}{dx} x^2 = 2x,$$

find the derivative of $y = \sqrt{x}$. Proceed in the same manner as we found the derivative of $y = \ln x$ knowing the derivative of $y = e^x$.

11. You want create a tin can which can hold a volume or $16\pi \,\mathrm{in}^3$, but with a minimum surface area (to minimize the cost of materials). What are the radius and height of the can which minimizes the surface area? (Recall: the volume and surface area of a cylinder are $\pi r^2 h$ and $2\pi r^2 + 2\pi r h$, respectively.)

Conceptual Problems, Exam 5:

12. Suppose $f(x) = e^x \sinh x$. Evaluate $f'(x)$ by (1) substituting the definition for $\sinh x$, simplifying, and differentiating, and (2) using the product rule, substituting definitions as appropriate, and then simplifying. Verify that your results are the same.

13. Suppose that f, g, and h are differentiable functions. Find

$$\frac{d}{dx} g(f(x))(h(x))^3.$$

14. Consider the function $g(x) = x^2 - 2\cos(x)$. Find all stationary points (if any), and inflection points (if any). (Hint: You know enough about graphing to analyze the first and second derivatives!)

Conceptual Problems, Exam 6:

15. Consider the curve implicitly defined by

$$ax^3 + y^4 = 1, \quad a \neq 0.$$

By using implicit differentiation, decide whether or not the curve has terrace points at $x = 0$.

(Note: We definite a terrace point as a point with $f''(x) = 0$, but without a local extremum at $(x, f(x))$.)

16. Evaluate

$$\lim_{x\to\infty} \frac{\frac{d}{dx}\operatorname{arcsinh}(x)}{\frac{d}{dx}\operatorname{arccosh}(x)},$$

and explain this limit geometrically.

17. Find $\frac{d}{dx}\operatorname{arccot}(x)$. (Hint: $\frac{d}{dx}\cot(x) = -\csc^2 x$. You should be able to derive any other trigonometric identities you need.)

Summary of needed facts about hyperbolic trigonometric functions:

- We define

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \quad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

- It easily follows that $\cosh^2(x) - \sinh^2(x) = 1$.
- The following derivatives are straightforward to calculate using the definitions:

$$\frac{d}{dx}\cosh(x) = \sinh(x), \quad \frac{d}{dx}\sinh(x) = \cosh(x).$$