True/False Oral Quiz on Limits and Continuity

BC Calculus I

3 March 2011

$$\lim_{x \to 0^-} \lfloor -x \rfloor = -1.$$

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Answer:

$$\lim_{x \to 0^-} \lfloor -x \rfloor = -1.$$

Answer:

False. The limit is 0.

$$\lim_{x\to\frac{\pi}{2}} \sec x \quad \text{DNE}(+\infty).$$

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Answer:

$$\lim_{x \to \frac{\pi}{2}} \sec x \quad \text{DNE}(+\infty).$$

Answer:

False. The limit DNE.

$f(x) = 3 - \dfrac{4}{x^2}$ is continuous on its domain.

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Answer:

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Answer:

True. Recall that f is not defined for $x = 0$.

$$\lim_{x \to 1^+} \ln(x-1) \quad \text{DNE}(-\infty).$$

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Answer:

$$\lim_{x \to 1^+} \ln(x-1) \quad \text{DNE}(-\infty).$$

Answer:

True.

$$\lim_{x \to \frac{\pi}{2}^-} \cot x \quad \text{DNE}(-\infty).$$

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Answer:

$$\lim_{x \to \frac{\pi}{2}^-} \cot x \quad \text{DNE}(-\infty).$$

Answer:

False. The limit is 0.

The function $f(x) = \lfloor 2x - 3 \rfloor$ has a removable discontinuity at $x = \dfrac{1}{2}$.

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Answer:

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Answer:

False. The discontinuity is essential.

$$\lim_{x \to -\infty} \frac{x^2 + 1}{x|x| - 1} = -1.$$

$$\lim_{x \to -\infty} \frac{x^2 + 1}{x|x| - 1} = -1.$$

Answer:

$$\lim_{x \to -\infty} \frac{x^2+1}{x|x|-1} = -1.$$

Answer:

True.

$$\lim_{x \to 1^+} \frac{(x-1)^2}{x^2-1} \quad \text{DNE}(+\infty).$$

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Answer:

$$\lim_{x \to 1^+} \frac{(x-1)^2}{x^2-1} \quad \text{DNE}(+\infty).$$

Answer:

False. The limit is $\displaystyle\lim_{x \to 1^+} \frac{x-1}{x+1} = 0.$

$y = \tan x$ is not continuous at $x = \dfrac{\pi}{2}$.

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Answer:

$y = \tan x$ is not continuous at $x = \dfrac{\pi}{2}$.

Answer:

False. $\dfrac{\pi}{2}$ is not in the domain of the function.

$$\lim_{x\to 0}\left(-\frac{1}{x^4}\right)\ \text{ DNE}(-\infty).$$

$$\lim_{x \to 0} \left(-\frac{1}{x^4} \right) \quad \text{DNE}(-\infty).$$

Answer:

$\lim_{x \to 0} \left(-\frac{1}{x^4} \right)$ DNE$(-\infty)$.

Answer:

True. $-\frac{1}{x^4}$ is always negative when $x \neq 0$.

$$\lim_{x \to 0^-} \csc x \quad \text{DNE}(+\infty).$$

$$\lim_{x\to 0^-} \csc x \quad \text{DNE}(+\infty).$$

Answer:

$$\lim_{x \to 0^-} \csc x \quad \text{DNE}(+\infty).$$

Answer:

False. The limit DNE($-\infty$).

The derivative of $f(x) = |x|$ is not defined at $x = 0$.

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Answer:

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Answer:

True.

$$\lim_{x \to \infty} -2e^{-x} = 0.$$

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Answer:

$$\lim_{x \to \infty} -2e^{-x} = 0.$$

Answer:

True.

$$\lim_{x \to \infty} \frac{3x^2 - x}{4x^2 - x^3} = \frac{3}{4}.$$

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Answer:

$$\lim_{x \to \infty} \frac{3x^2 - x}{4x^2 - x^3} = \frac{3}{4}.$$

Answer:

False. The limit is 0.

If a function is continuous on its domain, then it is differentiable on its domain.

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Answer:

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Answer:

False. Consider $f(x) = |x|$.

$$\lim_{x \to 0^-} \frac{\sin(3x)}{x} = 3.$$

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Answer:

$$\lim_{x \to 0^-} \frac{\sin(3x)}{x} = 3.$$

Answer:

True.

$$\lim_{x \to 0^+} \frac{x}{\tan(2x)} = 2.$$

$$\lim_{x \to 0^+} \frac{x}{\tan(2x)} = 2.$$

Answer:

$$\lim_{x \to 0^+} \frac{x}{\tan(2x)} = 2.$$

Answer:

False. The limit is $\frac{1}{2}$.

$\sec x$ has an essential discontinuity at $x = \dfrac{\pi}{2}$.

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Answer:

$\sec x$ has an essential discontinuity at $x = \dfrac{\pi}{2}$.

Answer:

False. $\sec x$ is not defined at $x = \frac{\pi}{2}$.

$$\lim_{x \to \frac{1}{2}^+} \lfloor 1 - 2x \rfloor = -1.$$

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Answer:

$$\lim_{x \to \frac{1}{2}^+} \lfloor 1 - 2x \rfloor = -1.$$

Answer:

True.

$$\lim_{x \to -\frac{\pi}{2}^+} \tan x \quad \text{DNE}(-\infty).$$

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Answer:

$$\lim_{x \to -\frac{\pi}{2}^+} \tan x \quad \text{DNE}(-\infty).$$

Answer:

True.

$f(x) = \dfrac{x^2 - 100}{x - 10}$ has a hole at $x = 10$.

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Answer:

$f(x) = \dfrac{x^2 - 100}{x - 10}$ has a hole at $x = 10$.

Answer:

True.

$$\lim_{x\to 0} f(x)g(x) = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x).$$

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Answer:

$$\lim_{x \to 0} f(x)g(x) = \lim_{x \to 0} f(x) \cdot \lim_{x \to 0} g(x).$$

Answer:

False. Consider $\lim_{x \to 0} \dfrac{\sin(x)}{x}$, and $\lim_{x \to 0} \sin(x)$ and $\lim_{x \to 0} \dfrac{1}{x}$.

$$\lim_{x \to -\infty} (x^3 - 4x + 1) \quad \text{DNE}(+\infty).$$

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Answer:

$$\lim_{x \to -\infty} (x^3 - 4x + 1) \;\; \text{DNE}(+\infty).$$

Answer:

False. The limit DNE($-\infty$).

$$\lim_{x \to 0^-} \frac{1 - \cos x}{x} = 0.$$

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Answer:

$$\lim_{x \to 0^-} \frac{1 - \cos x}{x} = 0.$$

Answer:

True.

$$\lim_{x \to -\infty} \frac{1 - x - x^2}{2 - x - x^3} = 0.$$

$$\lim_{x \to -\infty} \frac{1 - x - x^2}{2 - x - x^3} = 0.$$

Answer:

$$\lim_{x \to -\infty} \frac{1 - x - x^2}{2 - x - x^3} = 0.$$

Answer:

True.

$$\lim_{x\to 1^-} \frac{x^3+1}{x^3-1} \quad \text{DNE}(+\infty).$$

$$\lim_{x \to 1^-} \frac{x^3+1}{x^3-1} \quad \text{DNE}(+\infty).$$

Answer:

$$\lim_{x \to 1^-} \frac{x^3+1}{x^3-1} \quad \text{DNE}(+\infty).$$

Answer:

False. The limit DNE($-\infty$).