Shruthi Subramanian Due 16 May 2011

Reflection on Problem Writing

Throughout the course of this semester, I have come to understand the type of learner I am in math. For example, my usual strategy for studying or understanding math involves taking notes in class and doing the practice problems assigned for homework. If I still have trouble I usually do some extra problems from the book, watch some tutorials online or ask a friend. This semester, with the heavy emphasis on conceptual questions, I realized how very skills based by learning strategy was. This was clearly evident from our Fundays as I always had trouble on the conceptual section of the test. Though I understood and could complete the part of the question that required us to use our knowledge of the current topic, I could never connect to other topics in order to completely solve the problem. Creating my own conceptual original problems helped me cross this barrier. I have rarely been required to write my own problems in a math course and even if I did they were always skills based. This habit carried over at the start of the course and was the main problem I had on the first original problem assignment. Although the problems I wanted to write about were difficult for me to solve, they were not necessarily conceptual. I found I had a very hard time of thinking of questions on my own so I looked at other sources instead such as the book and recent Fundays. In the end, the problems I wrote about were almost exact replicas of the ones I had based them off of as I had trouble expanding upon the question to include other topics in math. I also had a hard time writing these questions because whenever I tried a something new, the problem ended up not making any sense. In addition, I found that even when I did try and re-create a problem, I still did not a clear conceptual understanding as

I made mistakes when I was solving my own problems. Overall, my first experience as an original problem writer did not go very well and I found I had a lot of improvements to make.

My second attempt was much better as I got a better understanding of what a conceptual question is. I focused on a topic that I knew was conceptual and weak in: proofs. Since we were working on finding the derivatives using the limit definition, I took a definition we had not proved in class and worked on it on my own for practice. I still had some trouble with my math along the way, but overall, I felt like I had improved as an original problem writer because I connected much more to other topics and incorporated it into my problems this time.

For our last original problem, I made the mistake of thinking a skills heavy based problem as a conceptual one. Although this one had less mathematical errors in the problems as we had to do multiple choice questions, it did not really incorporate other areas of math, a fact that I realized once we received the graded version back. In this sense, I feel as I did not really improve this time around as an original problem writer, but I did learn an important lesson.

Overall, I think I have improved as an original problem writer, but feel that I still have much more to learn. I can now identify between a skills heavy and a conceptual problem. I have also gotten better at making connections between topics that we cover in BC to ones that I have learned in previous math courses. Writing original problems greatly helps me better understand the lesson we are learning as I have to know all aspects of the subject rather than just the plug and chug skills version. Though I am no expert at writing original problems, I feel that with more practice and more time, I will be in the future.

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Mods 13-15

B

Problem 1

1. Motivation: I got this problem from looking through some problem in lesson 1.4 in our calculus book. I chose to write something similar because I found the original problem challenging and thought that by making a similar one, I would understand the concepts and process of how to solve such a problem easier.

2. Problem Statement: Given that m(0)= 2 and m'(x) ≥ 3 for all x ≤ 0. What can be said about m(-3)?
   These need to be the same!

3. Problem Solution: The ~~key fact~~ to solving this problem is to realize to that the speed limit law, which states f(b)- f(a) ≤ M(b-a), can be applied here. We are given that x ≤ 0 which can be rewritten as [a,0]. From here can substitute zero for 'b' and 3 for 'M' into our equation to get m(0) – m(a) ≤ 3(0-a). We are also given m(0) so we can further simplify this problem to 2 - m(a) ≤ -3a. This leads to the equation m(a) ≥ -3a – 2. With this equation, we can substitute -3 in for 'a' and solve. The answer is m(-3) ≥ 7.
   Use "-3" from the beginning.

4. Reflection: I am not a very good problem writer yet as I did not really modify the concept much from my reference. This is partly because I was not sure how to without having the problem make no sense. I hope to grow out of that as I start to write more of my own problems in the future.

Problem 2

1. Motivation: I got this problem from our first funday test. I am not good at proofs and thought the extra practice would help me greatly. ← This is a good idea

2. Problem Statement: Prove whether $f(x) = x^2 / \tan(x)$ is odd, even or neither.

3. Problem Solution: To start the proof, substitute '–x' into the equation. From here you get that $f(-x) = (-x)^2 / \tan(-x)$. Next, we can simplify this to $x^2 / -\tan(x)$ as $(-x)^2 = x^2$ and $\tan(-x) = -\tan(x)$. Now that $f(-x) = -f(x)$, we conclude that this function is odd.

4. Reflection: When I first started writing this problem, it was much more complicated. As I worked through it though, I found that my original problem did not really work out, though it may be due to the fact that I may have just messed up when I was trying to solve it. Either way, I learned through this process that writing your own problems takes a lot more thought than just plugging in numbers and hoping the solution will work out. It's kind of like when you are writing a book, you need to know how the story ends so as to incorporate the any clues that lead to the end throughout the book. Yes, this is correct

I know that writing problems is challenging! But try to bring in some new aspect to the problem. Also watch for errors!

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BC 1 original problems

Great first problem – but watch units on the second.

A –

1. **Motivation:** I am not that great at proofs nor I do I understand them very well, conceptually, when they are explained in graphical terms. Therefore, when we learned about the derivative of ln(x), I was very confused for a while. I wanted to try my hand at proving it algebraically and that's where this problem comes from.

Problem statement: Prove d/dx [ln(x)] = 1/x, using complete sentences.

Problem solution: Using the limit definition we get $\lim_{h\to0}[\frac{\ln(x+h)-\ln(x)}{h}]$. Using the log rule, log(a) - log(b) = log(a/b), we can simplify the numerator of our limit to get $\lim_{h\to0}[\frac{\ln(\frac{x+h}{x})}{h}]$. We can then factor out the denominator to get $\lim_{h\to0}\frac{1}{h}\ln(1+\left(\frac{h}{x}\right))$.

Watch parentheses.

Using another log rule, a log (b) = log (b^a), we then get the limit with the 1/h, as a power of the ln expression. From here, we need to make a few substitutions. To do this, let's define m=h/x, mx=h, and 1/h=1/mx. Now our limit expression looks like this;

$\lim_{m\to0}\ln(1+m)^{\frac{1}{mx}}$, which can further be simplified to $\lim_{m\to0}\ln\left((1+m)^{\frac{1}{m}}\right)^{\frac{1}{x}}$. The

1/x then becomes a co-efficient, and so we get $\left(\frac{1}{x}\right)\ln(\lim_{m\to0}(1+u)^{\frac{1}{u}})$. The limit part of this equation is actually equal to *e* as when you make the substitution m=1/n you get the compound interest formula for *e*, which is $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. Since this is *e,* we can

substitute this back into out equation to get (1/x) ln *e* and because ln *e* is 1, we are left with 1/x (Khan, 2011).

Reflection: I actually was not able to do this proof all on my own. I only got up to the substitution of m, on my own. After looking around online I found a really good website that explained it to me that I've cited above. Doing this proof showed me that I still need to practice more on proving definitions as I am able to get almost halfway before I get confused and don't know what to do. I hope to be able to improve on this in the future.

Good idea! I have never seen this proof before!

2. **Motivation:** When I was little I always used to get stuck on rate of change problems like finding the rate at which a car is moving or the how long it takes the boat to go upriver, etc. So even today, when I see an easy rate problem, my mind always registers it with a negative aspect. For example, this happened when I was doing one of the book problems so I thought I should write a similar one to help me get more comfortable with these types of problems. I know this might not be a conceptual question, but it is challenging for me so I decided to do it.

Watch units! Acceleration is not units of mph or mph^2

Problem statement: A car it traveling down the highway at 70 mph when it runs into traffic. The brakes are applied, giving the car a constant negative acceleration of 92mph^2. How long will it take the car to come to a stop and how far will it go before stopping?

Problem solution: From the problem statement we know that a(t) = -92 mph and that v(0) = 70 mph. From here we can solve for the DE of a(t) to get v(t) = -92t + 70. To find out how long the car will take to come to a stop, solve for t when v(t) =0. So -92t + 70 =0 so t= 92/70 or 46/35 seconds. This means it takes 46/35 seconds for the car to come to a standstill. To find how far it went in that time, we must solve for the distance equation by anti-differentiation. By doing this, we get p(t) = - 46t^2 + 70t + C. Since p(0) = 0 however, we also know that C=0. Since we already know it takes 46/35 seconds for the car to stop, we need to solve for p(46/35) to find out how far it went in that time. The equation for this looks like p(46/35) = -46 (46/35)^2 + 70 (46/35) and this turns out to be approximately 12.5 miles.

Reflection: The good thing is that I think I understand these rate problems a little better but I still need a lot more practice. The one hard thing I had with this problem was making all the numbers seen realistic. The very first few numbers I tried ended up coming out with a distance of 100 miles and they made absolutely no sense. I finally got some that actually worked and I hope that with more practice this will come more naturally to me.

Careful! Did the car really travel 12.5 miles in 46/35 seconds Does this make sense?

Think about how you could require a knowledge of linear approximations in a multiple choice problem. Also errors in chain rule in # 2! This seems a little too skills-heavy. More functions is not necessarily more conceptual

Original Problems 3

B

Motivation: We chose this problem because it presents a linear approximation problem in a more challenging way. Students must synthesize knowledge from previous math courses (such as finding equations from roots) with new knowledge.

there are infinitely many!

Problem Statement: Find the derivative of the cubic polynomial that has roots at $x = 0$, $x = -1$, and $x = 5$. Use linear approximation only!

You cannot force this issue on a multiple choice question

(A) $f'(x) = 3x^2 + 8x - 5$

(B) $f'(x) = 3x - 8$

(C) $f'(x) = 3x^2 - 8x + 5$

(D) $f'(x) = 3x^2 - 4x - 5$

(E) $f'(x) = 3x^2 - 8x - 5$

or $kx(x+1)(x-5)$, where $k \neq 0$

Problem Solution: To solve this problem, we must first find the function. If it the function has roots at these locations, then we can determine that the equation is $f(x) = x(x+1)(x-5)$, which expands to $f(x) = x^3-4x^2-5x$. Using the linear approximation method, we substitute $(x+h)$ for x. This expands to $f(x+h) = x^3 + 3x^2h + 3xh^2 + h^3 - 4x^2 - 4xh - 4h^2 - 5x - 5h$. In linear approximation, higher-order terms of h are trivial and can be ignored. Grouping like terms together leaves us with $f(x+h) = x^3 - 4x^2 - 5x + 3x^2h - 8xh - 5h$. If we factor out the h, we are left with $x^3 - 4x^2 - 5x + h(3x^2 - 8x - 5)$. In linear approximation, the first-order approximation is the derivative. Thus, we can conclude that $f'(x) = 3x^2 - 8x - 5$. The answer is choice (E).

Discussion: If students mixed up the signs of the roots and got the equation $f(x) = x(x+5)(x-1)$, then they would choose choice (A). If they forgot the root $x = 0$, then they would choose choice (B). If they forgot to distribute the minus sign in the expression $-5(x+h)$, they would choose choice (C). If they foiled $(x+h)^2$ incorrectly as $x^2 + xh + h^2$, they would choose choice (D).

Reflection: While solving this problem, we realized that our answer was different from what we found when we double-checked with the power rule. Going back through our work, we found that we had made some algebra mistakes! After fixing our errors, we realized that this was a perfect foundation for developing incorrect answer choices! We used our own mistakes to exploit those of others. When we first started writing the problem, we had trouble with finding a good function to use. The first function that we tried had imaginary solutions, so we could not give all the roots to solve the problem (we still wanted to maintain focus on linear approximation, not algebra skills). After a few attempts, however, we were able to find a suitable function.

Motivation: We modeled this problem after one of the conceptual questions we had on our most recent Funday. We expanded this concept by adding a quotient rule aspect to the problem. On first glance, problems similar to these might look overwhelming and difficult, but when examined closely, they are actually fairly systematic. We thought this question would be good practice to apply all the new formulas that we have recently learned.

Problem Statement: Suppose that f, g, h, a, and b are differentiable functions. Find $\dfrac{d}{dx}\dfrac{g(f(x))^3 h(2x+5)}{a(b(5x))}$.

$\frac{d}{dx} a(b(5x)) = a'(b(5x))\, b'(5x) \cdot 5$

(A) $\frac{(a(b(5x)))(h(2x+5))(3g(f(x))^2 f'(x))+(g(f(x))^3)(h'(2x+5)+2))-(g(f(x))^3 h(2x+5))(a'(b(5x))b'(5x))}{(a(b(5x)))^2}$

chain rule

(B) $\frac{(a(b(5x)))(h(2x+5))(3g(f(x))^2 g'(f(x))f'(x))+(g(f(x))^3)(h'(2x+5)+2))-(g(f(x))^3 h(2x+5))(a'(b(5x))b'(5x))}{(a(b(5x)))^2}$

(C) $\frac{(g(f(x))^3 h(2x+5))(a'(b(5x))b'(5x))-(a(b(5x)))(h(2x+5))(3g(f(x))^2 g'(f(x))f'(x))+(g(f(x))^3)(h'(2x+5)+2))}{(a(b(5x)))^2}$

(D) $\frac{(a(b(5x)))(h(2x+5))(3g(f(x))^2 g'(f(x))f'(x))+(g(f(x))^3)(h'(2x+5)+2))+(g(f(x))^3 h(2x+5))(a'(b(5x))b'(5x))}{(a(b(5x)))^2}$

(E) $\frac{(a(b(5x)))(h(2x+5))(3g(f(x))^2 g'(f(x))f'(x))+(g(f(x))^3)(h'(2x+5)+2))-(g(f(x))^3 h(2x+5))(a'(b(5x))b'(5x))}{25a^2b^2x^2}$

Problem Solution There are three main parts to this problem. The first component would be $g(f(x))^3$. The second component would be $h(2x+5)$ and the third component would be $a(b(5x))$. To solve this problem, it can be seen that one must use the chain rule within each component, then use the product rule between the first and second components and finally use the quotient rule numerator and the denominator. Start by working out the first component. Using the chain rule, we receive the result $3g(f(x))^2 g'(f(x))f'(x))$. Next, for the second component, again using the chain rule, we get $h'(2x+5)+2)$. Similarly, the third component is equal to $a'(b(5x))b'(5x)$. We can now use the product rule to simplify the numerator and you get $(h(2x+5))(3g(f(x))^2 g'(f(x))f'(x)) + (g(f(x))^3)(h'(2x+5)+2))$. The last step is to use the quotient rule to receive our final answer, which is $\frac{(a(b(5x)))(h(2x+5))(3g(f(x))^2 g'(f(x))f'(x))+(g(f(x))^3)(h'(2x+5)+2))-(g(f(x))^3 h(2x+5))(a'(b(5x))b'(5x))}{(a(b(5x)))^2}$.

NOT "+" $2h'(2x+5)$

Reflection: This biggest problem we faced while working out this problem was keeping all the parts in order and organized. Since there were so many parts to this problem if you try and do it all in your head, it becomes very difficult even if you know all the formulas. The first time we tried solving the problem we tried writing it in just one step but we found that we made many mistakes. To solve this issue, we split the question into components and dealt with each part one by one and then combined them all in the end. This made solving the problem much simpler, and more accurate as well.

Discussion: We know the correct answer is (B). If students made a mistake while differentiating $g(x^3)$, then $g'(f(x))$ would be missing from the final answer, and they would choose (A). If they made a mistake while using the quotient rule and switched uv' and $u'v$ (the order of the numerator), then they would choose (C). If they added instead of subtracted in the numerator while using the quotient rule, they would choose (D). If they were very keen to demonstrate algebra skills and foiled the denominator after the quotient rule, then they would choose (E).