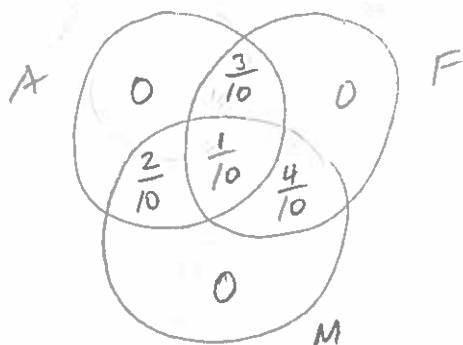


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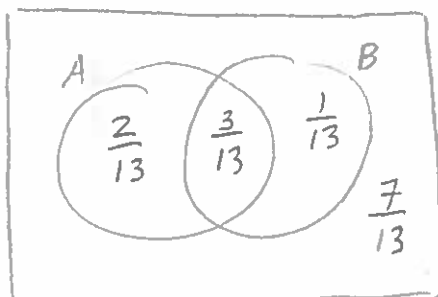
1. A student must choose at least two out of three electives: art, French, and mathematics. He chooses art with probability $\frac{3}{5}$, and art and French together with probability $\frac{2}{5}$. Also, he chooses all three electives with probability $\frac{1}{10}$. What is the probability that he chooses mathematics?



$$\frac{7}{10}$$

410

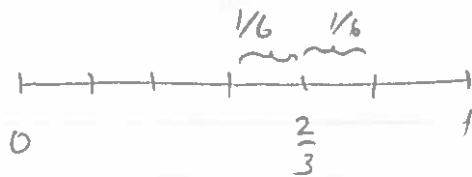
2. Let A and B be events such that $P(B) = \frac{4}{13}$, $P(\bar{A}) = \frac{8}{13}$, and $P(B - A) = \frac{1}{13}$. What is $P(A \cup B)$?



$$\frac{6}{13}$$

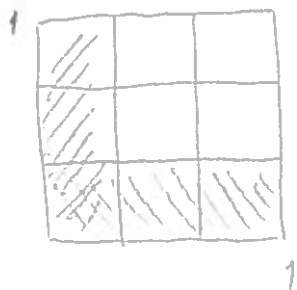
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3. Choose a number at random from the interval $[0, 1]$ with uniform density. Find the probability that $|B - \frac{2}{3}| < \frac{1}{6}$.



$$\frac{1}{3}$$

- +10 4. Choose independently two numbers B and C at random from the interval $[0, 1]$ with uniform density. Find the probability that $\min\{B, C\} \leq 1/3$.



$$\frac{5}{9}$$

- +10 5. You flip a coin twice; the probability that the coin comes up heads is $4/5$. Using generating functions, find the probability of obtaining exactly one head on two flips of the coin.

$$\left(\frac{1}{5} + \frac{4}{5}x\right)^2 = \frac{1}{25} + \frac{8}{25}x + \frac{16}{25}x^2$$

↑

$$\frac{8}{25}$$

+5 EXTRA CREDIT: You have a die with an undetermined number of sides. You know that every even number is twice as likely to come up as each odd number. The probability of rolling an odd number with one roll of the die is $17/50$. How many sides on the die?

Must be an odd number of sides

Evens: n faces, with probability $2p$

odds: $n+1$ faces, with probability p

$$n(2p) + (n+1)p = 1 \Rightarrow p = \frac{1}{3n+1}$$

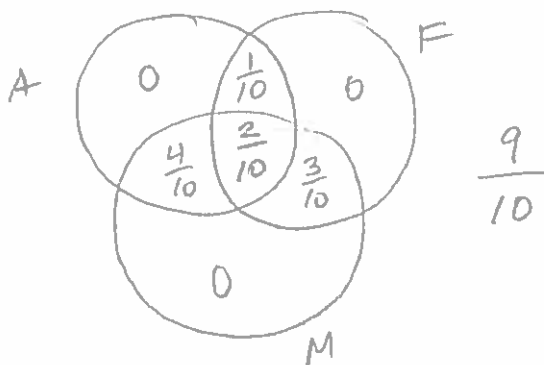
$$\frac{n+1}{3n+1} = \frac{17}{50} \Rightarrow 50n+50 = 51n+17$$

$$n = 33$$

So there are $33+34 = 67$ sides on the die.

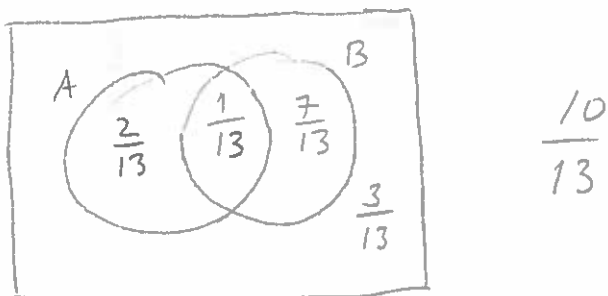
+10

1. A student must choose at least two out of three electives: art, French, and mathematics. He chooses art with probability $7/10$, and art and French together with probability $3/10$. Also, he chooses all three electives with probability $1/5$. What is the probability that he chooses mathematics?



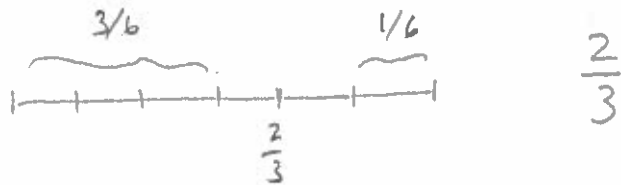
+10

2. Let A and B be events such that $P(B) = 8/13$, $P(\bar{A}) = 10/13$, and $P(B - A) = 7/13$. What is $P(A \cup B)$?

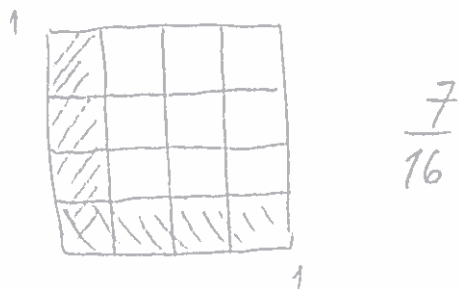


+10

3. Choose a number at random from the interval $[0, 1]$ with uniform density. Find the probability that $|B - 2/3| \geq 1/6$.



- +10 4. Choose independently two numbers B and C at random from the interval $[0, 1]$ with uniform density. Find the probability that $\min\{B, C\} \leq 1/4$.



- +10 5. You flip a coin twice; the probability that the coin comes up tails is $4/5$. Using generating functions, find the probability of obtaining exactly one head on two flips of the coin.

$$\left(\frac{4}{5} + \frac{1}{5}x\right)^2 = \frac{16}{25} + \frac{8}{25}x + \frac{1}{25}x^2$$

↑

$$\frac{8}{25}$$

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Odds: $n+1$ faces, with probability p

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$$\frac{n+1}{3n+1} = \frac{17}{50} \Rightarrow 50n + 50 = 51n + 17$$

$$n = 33$$

So there are $33 + 34 = 67$ sides on the die.