Homework Quiz 5A, 20 April 2018 Name:

You do not need to give exact answers. Leaving expressions like 7^{10} or ${}_{24}C_{19}$ is fine.

1. Find the number of three-of-a-kind hands in a 52-card deck (with no Joker).

$$\underbrace{13}_{\text{Choose rank 3OAK}} \times \underbrace{{}_4C_3}_{\text{Choose suits}} \times \underbrace{{}_{12}C_2}_{\text{choose ranks of last cards}} \times \underbrace{4^2}_{\text{choose suits of last cards}} = 54,912$$

2. Find the number of straights in a 52-card deck (with no Joker) which DO NOT contain a 7.

Can start with A, 2, 8, 9, 10.

$$\underbrace{5}_{\text{Choose starting card}} \times \underbrace{4^5}_{\text{choose suits}} - 20 \leftarrow \text{take out straight flushes}$$

$$= 5100$$

3. In how many different ways can the letters of the word "FEEDING" be arranged such that the vowels are all together?

EEI D F N G

$$\underbrace{5!}_{\text{arrange groups}} \times \underbrace{3}_{\text{EEI EIE or IEE}} = 360$$

4. In how many different ways can 5 girls and 5 boys form a circle such that the boys and the girls alternate?

← choose a girl

$$\underset{\substack{\text{other}\\\text{girls}}}{\underline{4!}} \times \underset{\substack{\text{seat}\\\text{the}\\\text{boys}}}{\underline{5!}} = 2880$$

5. How many groups of five people can be formed from a collection of 9 men and 6 women if the group must contain 2 women and 3 men?

$${}_9C_3 \times {}_6C_2 = 84 \times 15 = 1260$$

6. How many six-digit numbers begin with "37," have all different digits, and are even?

$$\frac{1}{3} \times \frac{1}{7} \times \underbrace{\underline{7} \times \underline{6} \times \underline{5}}_{\substack{\text{remaining}\\\text{different}\\\text{digits}}} \times \underset{\substack{\uparrow\\\text{Choose last}\\\text{digit 1ST!}}}{\underline{5}} = 1050$$

0, 2, 4, 6, or 8

EXTRA CREDIT: In how many ways can you write a string of 11 zeroes and 4 ones such that the ones are never adjacent?

Pair 1's with 0's to prevent adjacency.

10 10 10 10 0 0 0 0 0 0 0 ← ${}_{11}C_4$ arrangements

These all end in 0.

10 10 10 0 0 0 0 0 0 0 0 $\underline{1}$ ← ${}_{11}C_3$ arrangements

↑ end in 1

$${}_{11}C_4 + {}_{11}C_3 = {}_{12}C_4 = 495$$

Homework Quiz 5B, 20 April 2018 Name:

You do not need to give exact answers. Leaving expressions like 7^{10} or ${}_{24}C_{19}$ is fine.

1. Find the number of straights in a 52-card deck (with no Joker) which DO NOT contain an 8.

Can start with A, 2, 3, 9, 10

$$\underbrace{5}_{\text{choose starting card}} \times \underbrace{4^5}_{\text{choose suits}} - 20 \leftarrow \text{take out straight flushes}$$

$= 5100$

2. Find the number of three-of-a-kind hands in a 52-card deck (with no Joker).

$$\underbrace{13}_{\text{choose rank 3OAK}} \times \underbrace{{}_4C_3}_{\text{choose suits}} \times \underbrace{{}_{12}C_2}_{\text{choose ranks of last cards}} \times \underbrace{4^2}_{\text{choose suits of last cards}} = 54,912$$

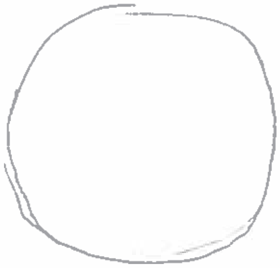
3. In how many different ways can the letters of the word "SEEMING" be arranged such that the vowels are all together?

EEI G M N S

$$\underbrace{5!}_{\text{arrange groups}} \times \underbrace{3}_{\text{EEI EIE}} = 360$$

4. In how many different ways can 6 girls and 6 boys form a circle such that the boys and the girls alternate?

G ← choose a girl



$$\underset{\text{other girls}}{5!} \times \underset{\text{seat the boys}}{6!} = 86{,}400$$

5. How many groups of five people can be formed from a collection of 8 men and 7 women if the group must contain 2 women and 3 men?

$${}_8C_3 \times {}_7C_2 = 56 \times 21 = 1176$$

6. How many six-digit numbers begin with "42," have all different digits, and are odd?

$$\underset{4}{\underline{1}} \times \underset{2}{\underline{1}} \times \underbrace{\underline{7} \times \underline{6} \times \underline{5}}_{\text{remaining different digits}} \times \underline{5} = 1050$$

↑ choose last digit first: 1, 3, 5, 7, or 9

Extra Credit: In how many ways can you write a string of 12 zeroes and 5 ones such that the ones are never adjacent?

Pair 1's with 0's to prevent adjacency:

10 10 10 10 10 0 0 0 0 0 0 0 ← ${}_{12}C_5$ arrangements

These all end in 0. ↗

10 10 10 10 0 0 0 0 0 0 0 0 $\underline{1}$ ← ${}_{12}C_4$ arrangements

↑ end in 1

$${}_{12}C_5 + {}_{12}C_4 = {}_{13}C_5 = 1287$$