1. Circle TRUE or FALSE for each question.

   (a) TRUE FALSE If $0° < D < 360°$, then a rotation in the $xy$-plane through $D$ has no real eigenvalues.

   (b) TRUE FALSE Any elementary matrix is also a symmetric matrix.

   (c) TRUE FALSE The geometric multiplicity and the algebraic multiplicity of an eigenvalue are always equal.

   (d) TRUE FALSE $\lambda = -1$ is an eigenalue for the projection onto the line $y = -2x$.

   (e) TRUE FALSE $\lambda = 1$ is an eigenvalue for the reflection across the line $y = -2x$.

2. Suppose a symmetric matrix with distinct eigenvalues has $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ as an eigenvector. Then another eigenvector for this matrix is:

   (a) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ (e) None of these.

3. Which of the following are eigenvectors of $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$? (There may be more than one answer.)

   (a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (e) None of these.

4. $p_{23}p_{13}$ is equal to:

   (a) $p_{12}$ (b) $p_{13}$ (c) $p_{23}$ (d) $I$ (e) None of these.

5. Consider the recurrence $a_{n+2} = 3a_{n+1} - 4a_n$, $a_0 = 1$, $a_1 = 3$. Which of the following matrices would you use to solve this recurrence with matrix methods?

   (a) $\begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -4 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 3 & -4 \end{bmatrix}$ (e) None of these.

6. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$.

7. You know that a recurrence with $a_0 = 0$ and $a_1 = 1$ has the form $a_n = c_1 \cdot 7^n + c_2 \cdot 3^n$. Find $c_1$ and $c_2$.

8. Find all $c, d$ for which the following system has infinitely many solutions.

$$\begin{aligned} 4x + y &= c \\ -8x + dy &= 14 \end{aligned}$$

9. Find the eigenvalues of $\begin{bmatrix} 2 & 1 \\ 7 & -4 \end{bmatrix}$.

10. Find the inverse of the matrix $\begin{bmatrix} 2 & -3 \\ -5 & 6 \end{bmatrix}$. Simplify as much as possible by multiplying out any fractions.

11. Suppose you know that

$$A = LDU = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}$$

and that $\det A = 12$. Which of $a$, $b$, and/or $c$ can you determine? Find the values of those you can determine.

12. The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ has eigenvalues of $\lambda = 3$ and $\lambda = -1$. The corresponding eigenvectors are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Use this to write $A = PDP^{-1}$ and calculate $A^3$.

13. You are solving the following system of equations:

$$\begin{aligned} 4x - 3y + 7z &= 10 \\ ax - 4y + 3x &= 5 \\ bx + 5y + z &= -9. \end{aligned}$$

Your first two steps involved elementary matrices $e_{21}^{-3}$ and $e_{31}^{1/2}$. What are $a$ and $b$?

14. If $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 3 \\ -1 & 2 & 1 \end{bmatrix}$ has an eigvenvalue of $\lambda = -2$, find a corresponding eigenvector.

15. What is your favorite color?

Extra Credit: Let $E$ be a linear transformation such that $E^2 = E$. What are the possible eigenvalues for $E$?