

1. Circle TRUE or FALSE for each question.

- (a) TRUE FALSE $\lambda = 1$ is an eigenvalue for the reflection across the line $y = -2x$.
(b) TRUE FALSE $\lambda = -1$ is an eigenvalue for the projection onto the line $y = -2x$.
(c) TRUE FALSE If $0^\circ < D < 360^\circ$, then a rotation in the xy -plane through D has no real eigenvalues.
(d) TRUE FALSE Any elementary matrix is also a symmetric matrix.
(e) TRUE FALSE The geometric multiplicity and the algebraic multiplicity of an eigenvalue are always equal.

2. Suppose a symmetric matrix with distinct eigenvalues has $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ as an eigenvector. Then another eigenvector for this matrix is:

- (a) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (e) None of these.

3. Which of the following are eigenvectors of $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$? (There may be more than one answer.)

- (a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (e) None of these.

4. $p_{12}p_{13}$ is equal to:

- (a) p_{12} (b) p_{13} (c) p_{23} (d) I (e) None of these.

5. Consider the recurrence $a_{n+2} = 2a_{n+1} - 3a_n$, $a_0 = 1$, $a_1 = 3$. Which of the following matrices would you use to solve this recurrence with matrix methods?

- (a) $\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ (e) None of these.

6. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

7. You know that a recurrence with $a_0 = 0$ and $a_1 = 1$ has the form $a_n = c_1 \cdot 8^n + c_2 \cdot 4^n$. Find c_1 and c_2 .

8. Find all c, d for which the following system has infinitely many solutions.

$$\begin{aligned}3x - y &= c \\ -6x + dy &= 12\end{aligned}$$

9. Find the eigenvalues of $\begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix}$.

10. Find the inverse of the matrix $\begin{bmatrix} 6 & -5 \\ -3 & 2 \end{bmatrix}$. Simplify as much as possible by multiplying out any fractions.

11. Suppose you know that

$$A = LDU = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}$$

and that $\det A = 10$. Which of a , b , and/or c can you determine? Find the values of those you can determine.

12. The matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has eigenvalues of $\lambda = 3$ and $\lambda = 1$. The corresponding eigenvectors are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Use this to write $A = PDP^{-1}$ and calculate A^3 .

13. You are solving the following system of equations:

$$4x - 3y + 7z = 10$$

$$ax - 4y + 3z = 5$$

$$bx + 5y + z = -9.$$

Your first two steps involved elementary matrices $e_{21}^{1/2}$ and e_{31}^{-2} . What are a and b ?

14. If $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 3 \\ -1 & 2 & 1 \end{bmatrix}$ has an eigenvalue of $\lambda = -2$, find a corresponding eigenvector.

15. What is your favorite color?

EXTRA CREDIT: Let E be a linear transformation such that $E^2 = E$. What are the possible eigenvalues for E ?