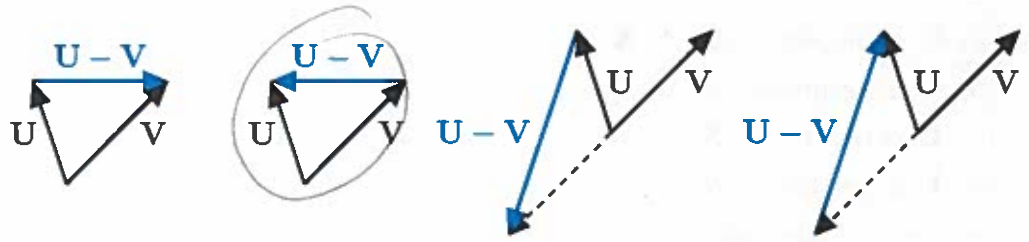
+3 1. Given vectors $\mathbf{U}$ and $\mathbf{V}$, which of the following graphically describes $\mathbf{U} - \mathbf{V}$? Circle your answer.



+10 2. Circle TRUE or FALSE for each question.

(a) TRUE (FALSE) Three distinct planes always meet in exactly one point.

(b) TRUE (FALSE) If a vector $\mathbf{U}$ is a scalar multiple of a vector $\mathbf{V}$, then $\mathbf{V}$ is a scalar multiple of $\mathbf{U}$.

(c) TRUE (FALSE) There are 48 direct symmetries of the cube.

(d) TRUE (FALSE) For any two matrices A and B, $AB = BA$.

(e) TRUE (FALSE) If two matrices A and B satisfy $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then it follows that either $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ or $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

+5 3. Suppose that two vectors $\mathbf{U}$ and $\mathbf{V}$ satisfy $\mathbf{U} \cdot \mathbf{V} = 42$. Then

(a) The angle between $\mathbf{U}$ and $\mathbf{V}$ is acute.

(b) The angle between $\mathbf{U}$ and $\mathbf{V}$ is $90°$.

(c) The angle between $\mathbf{U}$ and $\mathbf{V}$ is obtuse.

(d) None of the above.

+7 4. Let $\mathbf{U} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, let $\mathbf{X}$ be a **nonzero** vector, and put $\mathbf{Y} = P_{\mathbf{U}}(\mathbf{X})$. Circle ALL statements which might be true if an appropriate vector $\mathbf{X}$ is chosen. There may be more than one answer.

(a) $\mathbf{X} \cdot \mathbf{Y} = 0$.

(b) $\mathbf{X} \cdot \mathbf{Y} < 0$.

(c) $\mathbf{X}$ is parallel to $\mathbf{Y}$.

(d) $\mathbf{X} = \mathbf{Y}$.

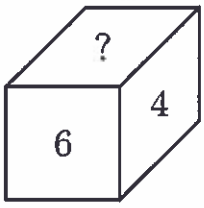
(e) None of the above.

25

+5 5. Let $\mathbf{X}$ and $\mathbf{Y}$ be a **nonzero** vectors which are not parallel, and put $\mathbf{U} = \mathbf{X} \times \mathbf{Y}$. Circle ALL true statements. There may be more than one answer.

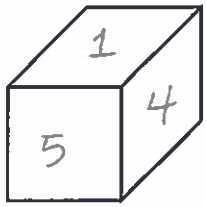
(a) $\mathbf{U}$ is perpendicular to $\mathbf{X}$.
(b) $\mathbf{U}$ is perpendicular to $\mathbf{Y}$.
(c) $\mathbf{U}$ is parallel to $\mathbf{X}$.
(d) $\mathbf{U}$ is parallel to $\mathbf{Y}$.
(e) None of the above.

+7 6. Write the matrix which transforms the die to the following position:



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

+7 7. Fill in the die after performing the transformation $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$.



+6 8. For which values of m are the vectors $\begin{pmatrix} m \\ 9 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ m \end{pmatrix}$ linearly dependent?

$$\frac{m}{4} = \frac{9}{m}$$

$$m^2 = 36$$

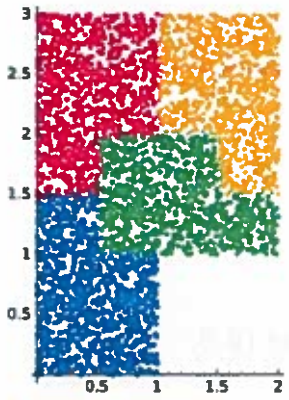
$$m = \pm 6$$

+6 9. Find the following matrix product: $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$.

$$\begin{bmatrix} -5 & 5 \\ -8 & 22 \end{bmatrix}$$

31

+8 10. Write the affine transformation, in matrix form, that describes the self-similarity of the green piece of the fractal (center).



Rotate 90°, shrink, and translate.

$$\begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

+8 11. Find the area of the triangle with vertices $(1, 2, 3)$, $(-1, 1, 1)$, and $(2, 1, -1)$.

(-2,-1,-2) → (-1,1,1)

(1,2,3)

(2,1,-1)

(1,-1,-4)

$(-2,-1,-2) \times (1,-1,-4) = (2,-10,3)$

$\frac{1}{2}\left|(2,-10,3)\right| = \frac{1}{2}\sqrt{113}$

+8 12. Find the distance from the vector $(1, 1, 1)$ to the plane $x + 2y + 3z = 6$.

0. The vector is in the plane!

24

+8 13. Give a parametric representation of the line through $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and parallel to $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4-2t \\ -1+3t \end{pmatrix}$$

$$x = 4-2t$$
$$y = -1+3t$$

+8 14. Find symmetric equations of the line through $(3, -1, 4)$ and $(6, 3, -7)$.

$$(3,-1,4) - (6,3,-7) = (-3,-4,11)$$

$$\frac{x-3}{-3} = \frac{y+1}{-4} = \frac{z-4}{11}, \text{ or}$$

$$\frac{x-6}{-3} = \frac{y-3}{-4} = \frac{z+7}{11}$$

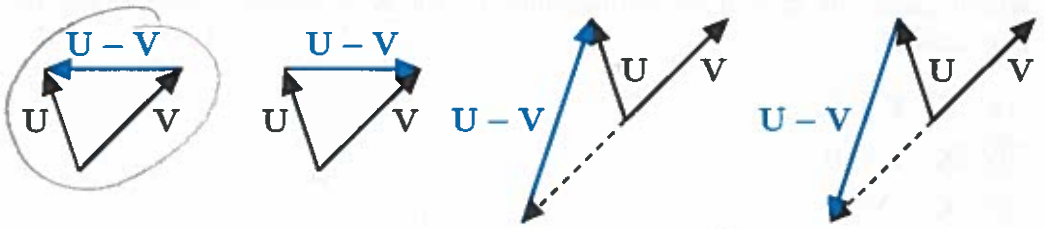
+4 15. What is your favorite food?

EXTRA CREDIT: How many vertices, edges, square faces, cubical cells, and four-dimensional hypercubes are there on a five-dimensional hypercube?

+5

20

+3 1. Given vectors $\mathbf{U}$ and $\mathbf{V}$, which of the following graphically describes $\mathbf{U} - \mathbf{V}$? Circle your answer.



+10 2. Circle TRUE or FALSE for each question.

(a) TRUE (FALSE) For any two matrices A and B, $AB = BA$.

(b) TRUE (FALSE) Three distinct planes always meet in exactly one point.

(c) TRUE (FALSE) There are 48 direct symmetries of the cube.

(d) TRUE (FALSE) If a vector $\mathbf{U}$ is a scalar multiple of a vector $\mathbf{V}$, then $\mathbf{V}$ is a scalar multiple of $\mathbf{U}$.

(e) TRUE (FALSE) If two matrices A and B satisfy $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then it follows that either $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ or $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

+5 3. Suppose that two vectors $\mathbf{U}$ and $\mathbf{V}$ satisfy $\mathbf{U} \cdot \mathbf{V} = 42$. Then

(a) The angle between $\mathbf{U}$ and $\mathbf{V}$ is obtuse.

(b) The angle between $\mathbf{U}$ and $\mathbf{V}$ is 90°.

(c) The angle between $\mathbf{U}$ and $\mathbf{V}$ is acute.

(d) None of the above.

+5 4. Let $\mathbf{X}$ and $\mathbf{Y}$ be a **nonzero** vectors which are not parallel, and put $\mathbf{U} = \mathbf{X} \times \mathbf{Y}$. Circle ALL true statements. There may be more than one answer.

(a) $\mathbf{U}$ is parallel to $\mathbf{X}$.

(b) $\mathbf{U}$ is parallel to $\mathbf{Y}$.

(c) $\mathbf{U}$ is perpendicular to $\mathbf{X}$.

(d) $\mathbf{U}$ is perpendicular to $\mathbf{Y}$.

(e) None of the above.

23

+7 5. Let $\mathbf{U} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, let $\mathbf{X}$ be a **nonzero** vector, and put $\mathbf{Y} = P_{\mathbf{U}}(\mathbf{X})$. Circle ALL statements which might be true if an appropriate vector $\mathbf{X}$ is chosen. There may be more than one answer.

(a) $\mathbf{X} \cdot \mathbf{Y} < 0$.

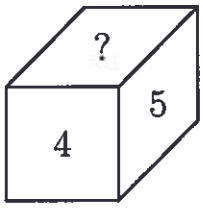
(b) $\mathbf{X} \cdot \mathbf{Y} = 0$.

(c) $\mathbf{X} = \mathbf{Y}$.

(d) $\mathbf{X}$ is parallel to $\mathbf{Y}$.

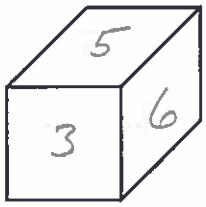
(e) None of the above.

+7 6. Write the matrix which transforms the die to the following position:



$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

+7 7. Fill in the die after performing the transformation $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$.



+6 8. For which values of m are the vectors $\begin{pmatrix} m \\ 25 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ m \end{pmatrix}$ linearly dependent?

$\frac{m}{4} = \frac{25}{m}$

$m^2 = 100$

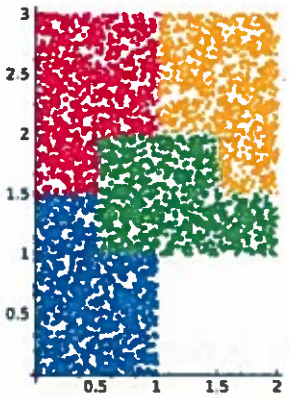
$m = \pm 10$

+6 9. Find the following matrix product: $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 4 \\ -3 & 2 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 6 \\ -13 & 22 \end{bmatrix}$$

33

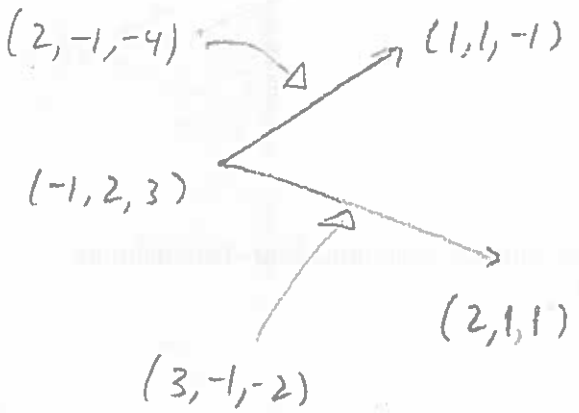
+8 10. Write the affine transformation, in matrix form, that describes the self-similarity of the orange piece of the fractal (upper right).



Reflect across the y-axis, shrink, and translate.

$$\begin{bmatrix} -1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 3/2 \end{pmatrix}$$

+8 11. Find the area of the triangle with vertices $(-1, 2, 3)$, $(1, 1, -1)$, and $(2, 1, 1)$.



$(2,-1,-4) \times (3,-1,-2)$
$= (-2,-8,1)$

$\frac{1}{2}\left|(-2,-8,1)\right| = \frac{1}{2}\sqrt{69}$

+8 12. Find the distance from the vector $(1, 1, 1)$ to the plane $3x + 2y + z = 6$.

0 The vector is in the plane!

24

+8 13. Give a parametric representation of the line through $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and parallel to $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2-3t \\ -5+2t \end{pmatrix}$$

$$x = 2 - 3t$$
$$y = -5 + 2t$$

+8 14. Find symmetric equations of the line through $(4, -2, 6)$ and $(7, 8, -3)$.

$$(4, -2, 6) - (7, 8, -3) = (-3, -10, 9)$$

$$\frac{x-4}{-3} = \frac{y+2}{-10} = \frac{z-6}{9} \quad \text{OR}$$

$$\frac{x-7}{-3} = \frac{y-8}{-10} = \frac{z+3}{9}$$

+4 15. What is your favorite food?

EXTRA CREDIT: How many vertices, edges, square faces, cubical cells, and four-dimensional hypercubes are there on a five-dimensional hypercube?

+5

20