

- +5 1. Find a vector  $\mathbf{v}$  such that the line  $5x - 8y = 0$  lies along  $\mathbf{v}$ .

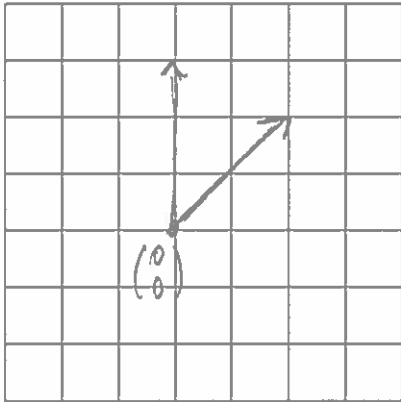
$$5x = 8y$$

$$y = \frac{5}{8}x$$

$$\begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

(or a vector parallel to this)

- +8 2. Find the angle between  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$  using the formula learned in class. Sketch!



$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{\begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right| \left| \begin{pmatrix} 0 \\ 3 \end{pmatrix} \right|}$$

$$= \frac{6}{\sqrt{8} \cdot 3} = \frac{6}{2\sqrt{2} \cdot 3} = \frac{1}{\sqrt{2}}$$

$$\text{So } \theta = 45^\circ \text{ or } \frac{\pi}{4}.$$

- +5 3. Find all  $x$  such that the vectors  $\begin{pmatrix} x \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 9 \\ x \end{pmatrix}$  are linearly dependent.

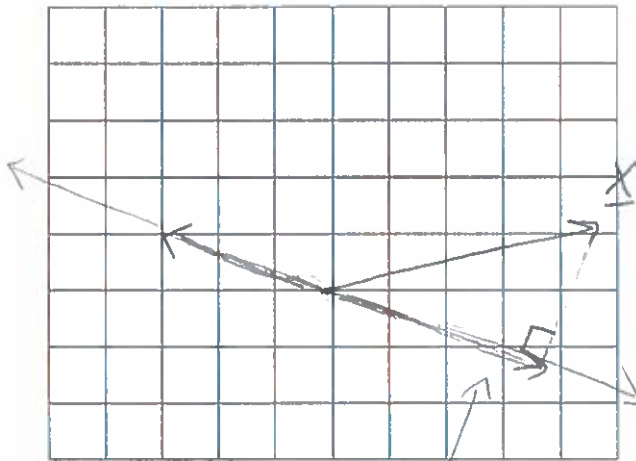
$$\frac{x}{9} = \frac{1}{x}$$

$$x^2 = 9$$

$$x = \pm 3$$

+10

4. Find a formula for the projection onto the line in the direction of  $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ . Sketch an example.



$$P_{\begin{pmatrix} -3 \\ 1 \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\begin{pmatrix} -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix}} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$= \frac{-3x + y}{10} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{9x - 3y}{10} \\ \frac{-3x + y}{10} \end{pmatrix} \text{ or}$$

$$P_{\begin{pmatrix} -3 \\ 1 \end{pmatrix}}(x)$$

$$\begin{bmatrix} \frac{9}{10} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{bmatrix} \text{ in matrix form}$$

+6

5. You are given that the projection onto the line in the direction of  $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$  is  $\begin{pmatrix} (16x - 4y)/17 \\ (-4x + y)/17 \end{pmatrix}$ . Find the reflection about this line. Simplify the algebra!

$$S(x) = 2P(x) - x = 2 \begin{pmatrix} \frac{16x - 4y}{17} \\ \frac{-4x + y}{17} \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \frac{32x}{17} - \frac{8x}{17} - x \\ -\frac{8x}{17} + \frac{2y}{17} - y \end{pmatrix} = \begin{pmatrix} \frac{15x}{17} - \frac{8x}{17} \\ -\frac{8x}{17} - \frac{15y}{17} \end{pmatrix}$$