Discrete Mathematics, Homework Quiz 5 18 April 2018

1. Simplify so that negation only appears right before the variables: $\neg(\neg A \vee (B \to \neg C))$.

DeMorgan's law: $A \wedge \neg(B \to \neg C)$

Rewrite $\to$ as $\vee$: $A \wedge \neg(\neg B \vee \neg C)$

DeMorgan's law: $A \wedge (B \wedge C)$

OR

$A \wedge B \wedge C$

2. Lena, Jerry, Chuck, and Errol were all playing blackjack. Lena was the dealer, and got a 17. If another player beats 17, they win, while if they tie or get lower, they lose. You know:

   (a) If Jerry lost or Chuck lost, then Errol also lost.

   (b) If Errol lost or Chuck won, then Jerry lost.

   (c) If Jerry won, then Chuck lost.

   (d) Chuck, Errol, and Jerry did not all lose.

   Who won?

Assume that Chuck lost. By (a), Errol lost, and then using (b), Jerry lost. All three losing contradicts (d). So, Chuck won.

Since Chuck won, Jerry lost from (b). Since Jerry lost, then Errol lost from (a).

3. Make a truth table for $P \vee (Q \to \neg P)$.

P	Q	$Q \to \neg P$	$P \vee (Q \to \neg P)$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

4. Prove one of the following. (For a challenge, prove choice (c).)

   (a) Prove that for all integers n, if $7n$ is odd, then n is odd.

   (b) Prove that $\sqrt{7}$ is irrational.

   (c) For all integers n, $n^5 - n$ is divisible by m. Find the largest m for which this statement is true, and prove it.

a) Suppose n is even. Then we may write $n = 2m$ for some $m \in \mathbb{Z}$. Then $7n = 7(2m) = 2(7m)$, and so $7n$ is even as well. (This is a proof of the contrapositive.)

b) Suppose $\sqrt{7}$ is rational. Then we can write, in lowest terms, $\sqrt{7} = \frac{a}{b}$. Then $a^2 = 7b^2$. Now since 7 is prime, this means that $7 | a$, so we may write $a = 7p$ for some $p \in \mathbb{Z}$. Thus $(7p)^2 = 7b^2$, and so $b^2 = 7p^2$. Similarly, we see that $7 | b$. But this means that a and b share a common factor of 7, contradicting $\frac{a}{b}$ being in lowest terms. Thus, $\sqrt{7}$ is irrational. (This a proof by contradiction.)