Discrete Mathematics, Homework Quiz 3 52 7 March 2018

1. Consider the arithmetic sequence $(a_n)_{n\geq 0}$ which starts $5, 12, 19, 26, \ldots$.

+2 (a) What is the next term in the sequence? 33

+3 (b) Find a formula for the nth term of this sequence.

$5 + 7n$

+5 (c) Find the sum of the first 50 terms of this sequence.

$5 + 12 + \cdots + 348$

$348 + \cdots + 5$

$353 \qquad 353$

$353 \cdot \frac{50}{2} = 8825$

2. Suppose that five terms of a geometric sequence with ratio $r = -\frac{1}{2}$ sum to 11. What is the first term of the sequence?

$$\frac{a_0\left(1-\left(\frac{-1}{2}\right)^5\right)}{1-\left(\frac{-1}{2}\right)} = a_0 \frac{\frac{33}{32}}{\frac{3}{2}} = a_0 \cdot \frac{33}{32} \cdot \frac{2}{3}$$

+5 +3

$$= \frac{11 a_0}{16} = 11 \Rightarrow a_0 = 16$$

3. Compute $\sum_{n=0}^{\infty} \frac{7^n}{4^{2n}}$.

$a = 1, \quad r = \frac{7}{16}, \quad \frac{a}{1-r} = \frac{1}{1-7/16}$

+2 +2 +2

$= \frac{16}{9}$ +1

4. Suppose an infinite geometric series has ratio $-\frac{3}{5}$ and sums to 4. What is the first term in this series?

$$\frac{a}{1-r} = \frac{a}{1-\left(\frac{-3}{5}\right)} = 4 \Rightarrow \frac{a}{\frac{8}{5}} = 4$$

+4

$$\Rightarrow a = \frac{32}{5}$$ +2

5. Use polynomial fitting to find a formula for the nth term of the sequence $(a_n)_{n\geq 0}$:

$2, 11, 24, 41, 62, \ldots$ $\quad 2n^2+7n+2.$

$9 \quad 13 \quad 17 \quad 21$

$4 \quad 4 \quad 4 \quad \leftarrow$ Quadratic +2

an^2+bn+c :

+6
$n=0$: $c=2$

$n=1$: $a+b+2=11$

$n=2$: $4a+2b+2=24$

$a+b=9 \quad b=9-a$

$4a+2b=22 \quad 4a+2(9-a)=22$ +3

$2a=4$

$a=2, \quad b=7$

6. Use polynomial fitting to *write the equations you would need* to find a formula for the nth term of the sequence $(a_n)_{n\geq 0}$: an^3+bn^2+cn+d

$1, 2, 6, 16, 35, 66, \ldots$

$1 \quad 4 \quad 10 \quad 19 \quad 31$

$3 \quad 6 \quad 9 \quad 12$

$3 \quad 3 \quad 3$ +2

+8
$n=0$: $d=1$

$n=1$: $a+b+c+d=2$

$n=2$: $8a+4b+2c+d=6$

$n=3$: $27a+9b+3c+d=35$ 16

+3 EXTRA CREDIT: Find a formula for the sequence given in (6).

Solve! $\frac{1}{2}(n^3+n+2)$

+1 some progress