

INCORRECT ANSWERS WITH NO WORK = NO PARTIAL CREDIT!

+8

1. Suppose you know for a fact that if Joe a CS major, then Alice is a math major.

(a) What can you conclude if Alice is an art major?

Joe is not a CS major. ✓

(b) What can you conclude if Alice is a math major?

No conclusion possible. ✓

2. For each of the statements below, give a domain of discourse for which the statement is true. Be sure that your domain of discourse is well-defined.

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(a) $\forall x \exists y (x \cdot y \neq 0)$.

$\{1, 2, 3, 4, \dots\}$

(b) $\forall x \forall y \forall z (x + y + z > 0)$.

$\{1, 2, 3, 4, \dots\}$

3. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10\}$, and $C = \{4, 5, 6, 7\}$. Also, assume you are working with the universe $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

+12

(a) Find \overline{B} .

$\{1, 3, 5, 7, 9\}$

(b) What is the cardinality of $B \times C$?

$5 \times 4 = 20$

(c) Find $A \setminus B$.

$\{1, 3, 5\}$

(d) Find $A \cap (B \cup C)$.

$\{2, 4, 5\}$

+12 4. Determine if each statement is true or false.

- (a) TRUE FALSE $f : \mathbb{N} \rightarrow \mathbb{N}, f(x) = 2x + 1$ is injective.
 (b) TRUE FALSE $f : \mathbb{N} \rightarrow \mathbb{Z}, f(x) = 2x + 1$ is injective.
 (c) TRUE FALSE $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 2x + 1$ is surjective.
 (d) TRUE FALSE $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 1$ is surjective.

+7 5. At the Easy Street Pizza Parlor, you can make a pizza by choosing one of four types of meat, one of five different vegetables, and one of three different cheeses. How many different pizzas can you order?

$$4 \times 5 \times 3 = 60$$

6. Let $S = \{1, 2, 3, 4, 5, 6\}$.

+3 (a) How many subsets are there in total?

$$2^6 = 64$$

+4 (b) How many subsets have $\{2, 3, 5\}$ as a subset?

8 (just add any subset of $\{1, 4, 6\}$)

+4 (c) How many subsets contain exactly one even number?

$$3 \times 8 = 24$$

\nearrow choose an even number \nwarrow choose a subset of $\{1, 3, 5\}$

+8 7. How many anagrams are there of the word "twitter" that start with the letter "e"?

$$e \text{ --- } \text{irtttw} \quad \frac{6!}{3!} = 120$$

8. How many ways are there to get a full house in poker?

$$+10 \quad \frac{13}{\text{rank for 3OAK}} \times \frac{\binom{4}{3}}{\text{suits for 3OAK}} \times \frac{12}{\text{rank for pair}} \times \frac{\binom{4}{2}}{\text{suits for pair}} = 3744$$

9. Out of 12 students, I need to select 4 for a team to enter a combinatorics competition! In addition, I need one student to be captain of the team.

+4 (a) How many ways can I do this if I first select the four students, and then select the captain from among them? (Do not just write an answer; write the appropriate combination/permutation you would use to make the calculation.)

$$\binom{12}{4} \times 4 = 1980$$

+4 (b) How many ways can I do this if I first select the caption from among the students, and the three others on the team? (Do not just write an answer; write the appropriate combination/permutation you would use to make the calculation.)

$$12 \times \binom{11}{3} = 1980$$

+2 (c) Are your answers to the previous two questions the same or different? Explain.

The same - they are counting the same thing in two different ways

10. You have ten identical Rubik's cubes, and you want to put them in three different boxes.

+3 (a) How many ways can you do this if there are no restrictions?

$$\text{Stars \& bars} = \binom{12}{2} = 66$$

+4 (b) How many ways can you do this if each box must contain at least one cube?

Put 1 in each box, then do stars & bars with 7 cubes and 3 boxes: $\binom{9}{2} = 36$

+7 (c) How many ways can you do this if no box can hold more than four cubes?

$$\binom{12}{2} - \left[\binom{3}{1} \binom{7}{2} - \binom{3}{2} \cdot 1 \right]$$

1 box has ≥ 5 cubes: stars & bars with 5 cubes

2 boxes have ≥ 5 cubes: 1

≥ 3 boxes with ≥ 5 cubes: 0

EXTRA CREDIT: You have 10 cookies which you want to give to four children named Ann, Bob, Cal, and Dee. However, you want to give Ann and Dee *at least* one cookie each, and you want to give Cal *more* cookies than Bob. In many ways can you do this?

Bob	Ann	Dee	Cal	stars / bars	
0	1	1	1	7	3 $\binom{9}{2} = 36$
1	1	1	2	5	3 $\binom{7}{2} = 21$
2	1	1	3	3	3 $\binom{5}{2} = 10$
3	1	1	4	1	3 $\binom{3}{2} = 1$

Cases on how many cookies Bob has. Give Cal 1 more, then stars & bars on just Ann, Dee, and Cal. 70