

1. Circle the letter before any sentence which is a mathematical statement.

(a) What time is it?

(b) $x + 10 = 5$.

(c) If n is an even integer, then $n + 1$ is also an even integer.

(d) This sentence is false.

(e) $\forall x \exists y (x + y = 12)$.

2. Consider the statement "If Frank has blue eyes, then Ellie has brown eyes." Assume that this statement is true.

(a) Suppose you know that Frank has green eyes. What can you conclude? Explain.

No conclusion. An implication is always true if the antecedent is false.

(b) Suppose you know that Ellie has blue eyes. What can you conclude? Explain.

Frank does not have blue eyes. This is the contrapositive.

3. Consider the statement "If Frank has blue eyes, then Ellie has brown eyes."

(a) Write the converse of this statement.

If Ellie has brown eyes, then Frank has blue eyes.

(b) Write the contrapositive of this statement.

If Ellie does not have brown eyes, then Frank does not have blue eyes.

4. Consider the statement $\forall x \exists y (x + y = \sqrt{2})$.

(a) Give a domain for which this statement is true. (No explanation needed.)

\mathbb{R}

(b) Give a domain for which this statement is false. (No explanation needed.)

\mathbb{N} (or \mathbb{Z} , \mathbb{Q} , or other possible answers).

5. Decide if the following are true or false. Circle one. Here, A and B are sets, and $C = \{0, 1, 2, 3, 4\}$. Assume U is the set of all integers.

- (a) TRUE **FALSE** If $A \subseteq B$, then $A \subset B$.
- (b) **TRUE** FALSE $C \subset \mathbb{N}$.
- (c) TRUE **FALSE** $C \subset C$.
- (d) TRUE **FALSE** $\{2, 3\} \in C$.
- (e) **TRUE** FALSE $\emptyset \subset C$.

6. Let $A = \mathbb{N}$, $B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $C = \{3, 6, 9, 12\}$, and $D = \{1, 3, 5, 7, 9\}$, with universe $U = \mathbb{Z}$. Find:

(a) $D \cap \bar{A}$

\emptyset

(b) $B \setminus D$

$\{0, 2, 4, 6, 8, 10\}$

(c) $\overline{B \cup D}$

$\{n \in \mathbb{Z} : n < 0 \text{ or } n > 10\} =$

(d) $A \cup \bar{A}$

$U = \mathbb{Z}$

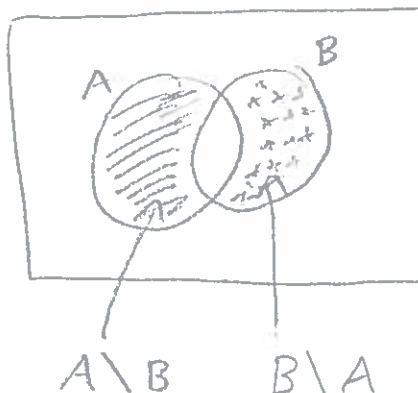
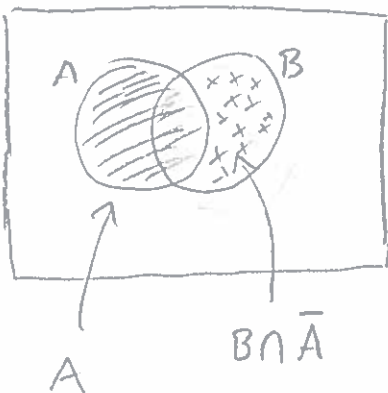
$\{\dots, -3, -2, -1, 11, 12, 13, \dots\}$

(e) $B \cap \overline{C \cup D}$

$\{0, 2, 4, 8, 10\}$

7. Using Venn diagrams, decide if the statement $A \cup (B \cap \bar{A}) = (A \setminus B) \cup (B \setminus A)$ is true or false.

U



False - shaded areas are not equal.