

SYMMETRIES OF THE CUBE

Let $\mathbf{R}_{1,1,1}$ be that symmetry which rotates the cube 120° clockwise around the axis through $(1, 1, 1)$ and $(-1, -1, -1)$. Let $\mathbf{R}_{1,y,1} = \mathbf{R}_{-1,y,-1}$ be that symmetry whose axis of rotation joins the midpoints of $(1, 1, 1)$ and $(1, -1, 1)$, and $(-1, 1, -1)$ and $(-1, -1, -1)$.

$$1. \mathbf{R}_{1,1,1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}: \text{Clockwise } 120^\circ \text{ rotation about the axis.}$$

$$2. \mathbf{R}_{1,1,1}^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}: \text{Counterclockwise } 120^\circ \text{ rotation about the axis.}$$

$$3. \mathbf{R}_{1,1,-1} = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}: \text{Clockwise } 120^\circ \text{ rotation about the axis.}$$

$$4. \mathbf{R}_{1,1,-1}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}: \text{Counterclockwise } 120^\circ \text{ rotation about the axis.}$$

$$5. \mathbf{R}_{1,-1,1} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}: \text{Clockwise } 120^\circ \text{ rotation about the axis.}$$

$$6. \mathbf{R}_{1,-1,1}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}: \text{Counterclockwise } 120^\circ \text{ rotation about the axis.}$$

$$7. \mathbf{R}_{-1,1,1} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}: \text{Clockwise } 120^\circ \text{ rotation about the axis.}$$

$$8. \mathbf{R}_{-1,1,1}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}: \text{Counterclockwise } 120^\circ \text{ rotation about the axis.}$$

$$9. \mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}: \text{Clockwise } 90^\circ \text{ rotation about the } x\text{-axis.}$$

$$10. \mathbf{R}_x^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}: 180^\circ \text{ rotation about the } x\text{-axis.}$$

11. $\mathbf{R}_x^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$: Counterclockwise 90° rotation about the x -axis.

12. $\mathbf{R}_y = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$: Clockwise 90° rotation about the y -axis.

13. $\mathbf{R}_y^2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$: 180° rotation about the y -axis.

14. $\mathbf{R}_y^3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$: Counterclockwise 90° rotation about the y -axis.

15. $\mathbf{R}_z = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: Clockwise 90° rotation about the z -axis.

16. $\mathbf{R}_z^2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: 180° rotation about the z -axis.

17. $\mathbf{R}_z^3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: Counterclockwise 90° rotation about the z -axis.

18. $\mathbf{R}_{x,1,1} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$: 180° rotation about the axis.

19. $\mathbf{R}_{x,1,-1} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$: 180° rotation about the axis.

20. $\mathbf{R}_{1,y,1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$: 180° rotation about the axis.

21. $\mathbf{R}_{1,y,-1} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$: 180° rotation about the axis.

22. $\mathbf{R}_{1,1,z} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$: 180° rotation about the axis.

23. $\mathbf{R}_{1,-1,z} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$: 180° rotation about the axis.

24. $\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: The identity.