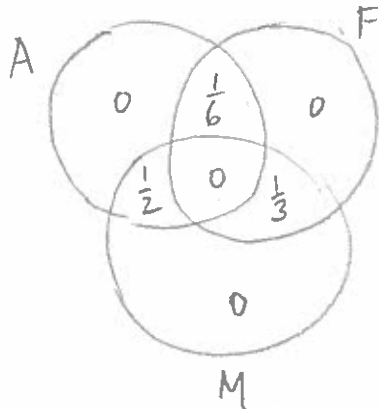


1. A student must choose exactly two out of three electives: art, French, and mathematics. He chooses art with probability  $\frac{2}{3}$ , and art and French together with probability  $\frac{1}{6}$ . What is the probability that he chooses mathematics?



$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

OR

$$1 - \frac{1}{6} = \frac{5}{6} \text{ (not art and French)}$$

2. Let  $A$  and  $B$  be events such that  $P(B) = \frac{3}{11}$ ,  $P(\bar{A}) = \frac{7}{11}$ , and  $P(A \cap B) = \frac{1}{11}$ . What is  $P(A \cup B)$ ?

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{4}{11} + \frac{3}{11} - \frac{1}{11} \\ &= \frac{6}{11} \end{aligned}$$

3. Choose a number at random from the interval  $[0, 1]$  with uniform density. Find the probability that  $|\frac{3}{4} - B| < \frac{1}{4}$ .

$$-\frac{1}{4} < \frac{3}{4} - B < \frac{1}{4}$$

$$-1 < -B < -\frac{1}{2}$$

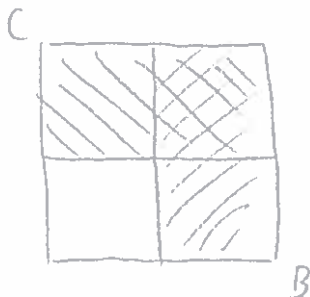
$$1 > B > \frac{1}{2}$$

$$\frac{1}{2} < B < 1$$

$$\text{Probability} = \frac{1}{2}$$

4. Choose independently two numbers  $B$  and  $C$  at random from the interval  $[0, 1]$  with uniform density. Find the probability that  $\max\{B, C\} > 1/2$ .

$\max\{B, C\} > \frac{1}{2}$  means  $B > \frac{1}{2}$  OR  $C > \frac{1}{2}$  (maybe both)



↑  
union of all areas.

$$\text{Probability} = \frac{3}{4}$$

5. You have two 12-sided dice, each with the numbers 1–12 on them. Each face is equally likely to come up. How would you find the probability of rolling a 10 if you rolled two dice? Your answer must include generating functions!

Multiply out  $(x^1 + x^2 + x^3 + \dots + x^{11} + x^{12})^2$  and look for the coefficient of  $x^{10}$ .

**EXTRA CREDIT:** You have a 100-sided die, and each of the faces 1–100 is equally likely to be rolled. You roll the die twice. What is the probability that your second roll is higher than your first? Write your answer as a fraction in lowest terms!

Not quite  $\frac{1}{2}$ .... you might roll the same number twice.

This happens in 100 ways, so we subtract these first.

$$\frac{\frac{1}{2}(10,000 - 100)}{10,000} = \frac{\frac{1}{2}(9900)}{10,000} = \frac{99}{200}$$