Solutions, Midterm II   7 April 2017

1 $\lambda = 1 : \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ The x-axis does not move.

There is a $90°$ rotation in the yz-plane ($y \to z$, $z \to -y$),

so no eigenvalues / eigenvectors!

2 $(\lambda+1)(\lambda-3) = 0 \Rightarrow$ Characteristic equation is $\lambda^2 - 2\lambda - 3 = 0$

$\det \begin{bmatrix} 1-\lambda & b \\ c & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - bc = \lambda^2 - 2\lambda + 1 - bc = 0 \Rightarrow 1 - bc = -3$

So any values of b, c such that $bc = 4$ will work.

3 $\begin{bmatrix} 4 & 0 & 1 \\ -3 & 3 & 5 \\ 0 & 0 & 7 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{aligned} 4x + z &= 4x \\ -3x + 3y + 5z &= 4y \\ 7z &= 4z \Rightarrow z = 0 \end{aligned}$

Since $z = 0$, the 2^{nd} equation becomes $y = -3x$: eigenvector is $\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$

4 From the geometry of the projection: $\lambda = 0 : \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; $\lambda = 1 : \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

5 $P = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

$A^5 = PD^5P^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 32 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 32 & 0 \end{bmatrix} = \begin{bmatrix} 32 & 0 \\ 32 & 0 \end{bmatrix}$

6 $\det \begin{bmatrix} 6-\lambda & -4 & 0 \\ 0 & 2-\lambda & 3 \\ 2 & 0 & -8-\lambda \end{bmatrix} \begin{matrix} 6-\lambda & -4 \\ 0 & 2-\lambda \\ 2 & 0 \end{matrix} = 0$

$$(6-\lambda)(2-\lambda)(-8-\lambda) - 24 = 0$$

7 $A \begin{pmatrix} a_n \\ a_{n+1} \end{pmatrix} = \begin{pmatrix} a_{n+1} \\ a_{n+2} \end{pmatrix} \Rightarrow A = \begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix}$ $\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

From recurrence relation ↵   ↳ From initial values

8 $\text{Det} \begin{bmatrix} -5 & 6 \\ -7 & 8 \end{bmatrix} = -40 - (-42) = 2$

$\begin{bmatrix} -5 & 6 \\ -7 & 8 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 8 & -6 \\ 7 & -5 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 7/2 & -5/2 \end{bmatrix}$

9 Since this is an elementary matrix, the inverse is $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10 To have no solutions, the equations must represent parallel lines. Thus, $b = 4$. The lines are the same when $a = -5$. So any value for a is fine as long as $a \neq -5$.

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11 $M^T = \begin{bmatrix} 6 & 2 & -1 \\ -3 & 9 & 3 \\ 4 & 0 & 8 \end{bmatrix}$. Just interchange columns and rows.

12 P_{13}: switch rows 1 and 3 $\rightarrow \begin{matrix} \text{row } 3 \\ \text{row } 2 \\ \text{row } 1. \end{matrix}$ Then P_{23} switches rows 2 and 3 of this result: $\begin{matrix} \text{row } 3 \\ \text{row } 1 \\ \text{row } 2 \end{matrix}$. Now switch rows 1 and 2: $\begin{matrix} \text{row } 1 \\ \text{row } 3 \\ \text{row } 2 \end{matrix}$

Thus, in the end, only rows 2 and 3 are switched: P_{23} does this.

13 Since symmetric matrices with distinct eigenvalues have orthogonal eigenvectors, any vector orthogonal to $\binom{-2}{3}$ is fine, such as $\binom{3}{2}$.

14 Using this relationship to write $A = LDU$, we have $D = \begin{bmatrix} 4 & 0 \\ 0 & -1/2 \end{bmatrix}$

Since $\det A = \det D$, then $\det A = 4(-\frac{1}{2}) = -2$

15
$$\begin{aligned} 2x + 2y - 3z &= 7 \\ 3y - 5z &= 8 \\ 6y - 6z &= -4 \end{aligned}$$
Next, use e_{32}^{-2} to eliminate the "$6y$".
$$\begin{aligned} 2x + 2y - 3z &= 7 \\ 3y - 5z &= 8 \\ 4z &= -20 \end{aligned}$$
Then get 1 coefficients by using $D = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

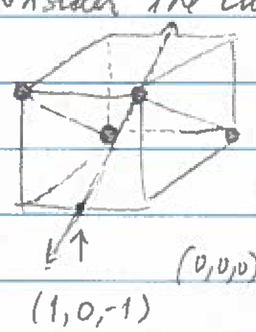
Therefore, we have $S_1 = e_{32}^{-2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$, $S_2 = D$

16 d comes from the 1,2 position of the expansion by minors

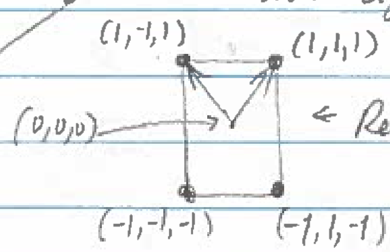
$\begin{bmatrix} x & x & x \\ -3 & x & 2 \\ 1 & x & 2 \end{bmatrix} \rightarrow -3(2) - (1)(2) = -8$

So $d = -(\frac{1}{7})(-8) = \frac{8}{7}$

Extra Credit. Consider the cube with corners $(\pm 1, \pm 1, \pm 1)$



Since this is an axis (which doesn't move), we get $\lambda = 1$ with eigenvector $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.



← Rectangle orthogonal to this axis

$\lambda = -1$: $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$