Please note that many of these questions need very little computation. Think before you just start trying a method which will take you a long time to complete!

1. Find all eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$.

2. Find numbers b and c such that $\begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix}$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$. **DO NOT** find the eigenvectors! (You only need one pair of values for b and c.)

3. The matrix $\begin{bmatrix} 4 & 0 & 1 \\ -3 & 3 & 5 \\ 0 & 0 & 7 \end{bmatrix}$ has an eigvenvalue of $\lambda = 4$. Find a corresponding eigenvector.

4. Find all eigenvalues and eigenvectors of the transformation P which is the projection onto the x-axis.

5. The matrix $A = \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}$ has eigenvalues of $\lambda = 0$ and $\lambda = 2$. The corresponding eigenvectors are $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Use this to write $A = PDP^{-1}$ and calculate A^5.

6. Find the characteristic equation of the matrix $\begin{bmatrix} 6 & -4 & 0 \\ 0 & 2 & 3 \\ 2 & 0 & -8 \end{bmatrix}$. You do not have to simplify the equation!

7. Consider the following recurrence:

$$a_{n+2} = 3a_{n+1} + 10a_n, \qquad a_0 = 6, \quad a_1 = 7.$$

   Find a matrix A and numbers c and d such that

$$\begin{pmatrix} a_n \\ a_{n+1} \end{pmatrix} = A^n \begin{pmatrix} c \\ d \end{pmatrix}.$$

8. Find the inverse of the matrix $\begin{bmatrix} -5 & 6 \\ -7 & 8 \end{bmatrix}$. Simplify as much as possible by multiplying out any fractions.

9. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

10. Find all a, b for which the system

$$\begin{aligned} x + 2y &= a \\ -2x - by &= 10 \end{aligned}$$

   has no solutions.

11. Suppose $M = \begin{bmatrix} 6 & -3 & 4 \\ 2 & 9 & 0 \\ -1 & 3 & 8 \end{bmatrix}$. Find M^T, the transpose of M.

12. Find a matrix M such that for any matrix A, we have $MA = P_{12}P_{23}P_{13}A$.

13. Suppose a symmetric matrix has eigenvalues $\lambda = -7$ and $\lambda = 12$. An eigenvector corresponding to $\lambda = -7$ is $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Give an eigenvector corresponding to $\lambda = 12$.

14. If

$$\begin{bmatrix} 1 & -1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5/2 & 1 \end{bmatrix} A = I,$$

find the determinant of A.

15. You are finding the LDU decomposition of a 3×3 matrix A. After the first two steps, you know so far that the matrix $e_{31}^{3} e_{21}^{-1/2} A$ results in the system

$$\begin{aligned} 2x + 2y - 3z &= 7 \\ 3y - 5z &= 8 \\ 6y - 6z &= -4. \end{aligned}$$

What are the next two matrices you will need to compute? In other words, as you continue to $S_2 S_1 e_{31}^{3} e_{21}^{-1/2} A$, what are S_1 and S_2?

16. Suppose

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -3 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}, \qquad A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

Given that $\det A = 7$, find d.

17. What is your favorite color?

Extra Credit: Consider the transformation M which performs a 180° rotation about the line passing through the points $(0, 0, 0)$ and $(1, 0, -1)$. Find all eigenvalues and eigenvectors of this transformation.