1. Write the affine transformation $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}5+2y-7x\\3-x-y\end{pmatrix}$ using its linear part and its translation.

$$f(x,y) = \begin{bmatrix}-7 & 2\\-1 & -1\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}5\\3\end{pmatrix}$$

2. If $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-y\\-x\end{pmatrix}$, compute $f \circ f\begin{pmatrix}x\\y\end{pmatrix}$. Give a *geometrical* explanation of your answer – in other words, explain how you could have arrived at your answer without doing any algebra, but just by thinking about what f does geometrically.

$$f\circ f\begin{pmatrix}x\\y\end{pmatrix} = f\begin{pmatrix}-y\\-x\end{pmatrix} = \begin{pmatrix}-(-x)\\-(-y)\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

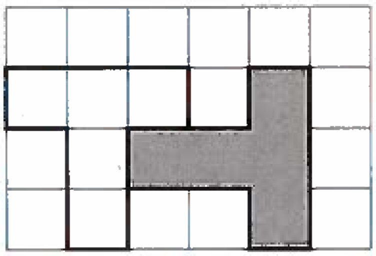
f actually describes a reflection about the line $y=-x$, and so performing a reflection twice takes you back where you started.

3. If $A = \begin{bmatrix}2 & 4\\-4 & -8\end{bmatrix}$, find B such that $BA = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$, but $B \neq \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$.

Many answers are possible. For example,

$$\begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}2 & 4\\-4 & -8\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

4. Write an affine transformation which transforms the white shape into the gray shape. Assume that the origin is at the lower left corner of the grid.



Rotate 90° CW, but then move $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Be careful!

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

5. Write the affine transformation which moves down 5 units, then reflects across the y-axis, and then moves to the left 10 units.

Down 5 units: $\begin{pmatrix} x \\ y-5 \end{pmatrix}$

Reflect across y-axis: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{pmatrix} x \\ y-5 \end{pmatrix} = \begin{pmatrix} -x \\ y-5 \end{pmatrix}$

Left 10 units: $\begin{pmatrix} -x-10 \\ y-5 \end{pmatrix}$

6. Find x such that $\det \begin{bmatrix} 2 & -3 \\ x & 5 \end{bmatrix} = 42$.

$2 \cdot 5 - x(-3) = 42$

$10 + 3x = 42 \Rightarrow x = \frac{32}{3}$