Exercises for Section 11.4

See www.CalcChat.com for worked-out solutions to odd-numbered exercis

In Exercises 1–6, find the cross product of the unit vectors and sketch your result.

1. $\mathbf{j} \times \mathbf{i}$
2. $\mathbf{i} \times \mathbf{j}$
3. $\mathbf{j} \times \mathbf{k}$
4. $\mathbf{k} \times \mathbf{j}$
5. $\mathbf{i} \times \mathbf{k}$
6. $\mathbf{k} \times \mathbf{i}$

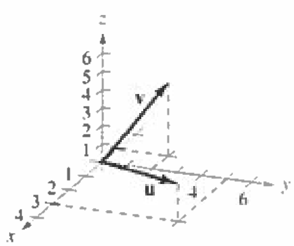
In Exercises 7–10, find (a) $\mathbf{u} \times \mathbf{v}$, (b) $\mathbf{v} \times \mathbf{u}$, and (c) $\mathbf{v} \times \mathbf{v}$.

7. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$
   $\mathbf{v} = 3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$
8. $\mathbf{u} = 3\mathbf{i} + 5\mathbf{k}$
   $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
9. $\mathbf{u} = \langle 7, 3, 2 \rangle$
   $\mathbf{v} = \langle 1, -1, 5 \rangle$
10. $\mathbf{u} = \langle 3, -2, -2 \rangle$
    $\mathbf{v} = \langle 1, 5, 1 \rangle$

In Exercises 11–16, find $\mathbf{u} \times \mathbf{v}$ and show that it is orthogonal to both u and v.

11. $\mathbf{u} = \langle 2, -3, 1 \rangle$
    $\mathbf{v} = \langle 1, -2, 1 \rangle$
12. $\mathbf{u} = \langle -1, 1, 2 \rangle$
    $\mathbf{v} = \langle 0, 1, 0 \rangle$
13. $\mathbf{u} = \langle 12, -3, 0 \rangle$
    $\mathbf{v} = \langle -2, 5, 0 \rangle$
14. $\mathbf{u} = \langle -10, 0, 6 \rangle$
    $\mathbf{v} = \langle 7, 0, 0 \rangle$
15. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$
    $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$
16. $\mathbf{u} = \mathbf{i} + 6\mathbf{j}$
    $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$

Think About It **In Exercises 17–20, use the vectors u and v shown in the figure to sketch a vector in the direction of the indicated cross product in a right-handed system.**



17. $\mathbf{u} \times \mathbf{v}$
18. $\mathbf{v} \times \mathbf{u}$
19. $(-\mathbf{v}) \times \mathbf{u}$
20. $\mathbf{u} \times (\mathbf{u} \times \mathbf{v})$

In Exercises 21–24, use a computer algebra system to find $\mathbf{u} \times \mathbf{v}$ and a unit vector orthogonal to u and v.

21. $\mathbf{u} = \langle 4, -3.5, 7 \rangle$
    $\mathbf{v} = \langle -1, 8, 4 \rangle$
22. $\mathbf{u} = \langle -8, -6, 4 \rangle$
    $\mathbf{v} = \langle 10, -12, -2 \rangle$
23. $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$
    $\mathbf{v} = \frac{1}{2}\mathbf{i} - \frac{3}{4}\mathbf{j} + \frac{1}{10}\mathbf{k}$
24. $\mathbf{u} = \frac{2}{3}\mathbf{k}$
    $\mathbf{v} = \frac{1}{2}\mathbf{i} + 6\mathbf{k}$

25. ***Programming*** Given the vectors u and v in component form, write a program for a graphing utility in which the output is $\mathbf{u} \times \mathbf{v}$ and $\|\mathbf{u} \times \mathbf{v}\|$.

26. ***Programming*** Use the program you wrote in Exercise 25 to find $\mathbf{u} \times \mathbf{v}$ and $\|\mathbf{u} \times \mathbf{v}\|$ for $\mathbf{u} = \langle -2, 6, 10 \rangle$ and $\mathbf{v} = \langle 3, 8, 5 \rangle$.

Area **In Exercises 27–30, find the area of the parallelogra[m] that has the given vectors as adjacent sides. Use a comput[er] algebra system or a graphing utility to verify your result.**

27. $\mathbf{u} = \mathbf{j}$
    $\mathbf{v} = \mathbf{j} + \mathbf{k}$
28. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$
    $\mathbf{v} = \mathbf{j} + \mathbf{k}$
29. $\mathbf{u} = \langle 3, 2, -1 \rangle$
    $\mathbf{v} = \langle 1, 2, 3 \rangle$
30. $\mathbf{u} = \langle 2, -1, 0 \rangle$
    $\mathbf{v} = \langle -1, 2, 0 \rangle$

Area **In Exercises 31 and 32, verify that the points are th[e] vertices of a parallelogram, and find its area.**

31. $(1, 1, 1)$, $(2, 3, 4)$, $(6, 5, 2)$, $(7, 7, 5)$
32. $(2, -3, 1)$, $(6, 5, -1)$, $(3, -6, 4)$, $(7, 2, 2)$

Area **In Exercises 33–36, find the area of the triangle with th[e] given vertices. (*Hint:* $\frac{1}{2}\|\mathbf{u} \times \mathbf{v}\|$ is the area of the triangle havin[g] u and v as adjacent sides.)**

33. $(0, 0, 0)$, $(1, 2, 3)$, $(-3, 0, 0)$
34. $(2, -3, 4)$, $(0, 1, 2)$, $(-1, 2, 0)$
35. $(2, -7, 3)$, $(-1, 5, 8)$, $(4, 6, -1)$
36. $(1, 2, 0)$, $(-2, 1, 0)$, $(0, 0, 0)$

37. ***Torque*** A child applies the brakes on a bicycle by applying a downward force of 20 pounds on the pedal when the cran[k] makes a 40° angle with the horizontal (see figure). The crank is 6 inches in length. Find the torque at P.

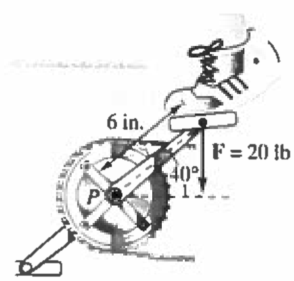


Figure for 37

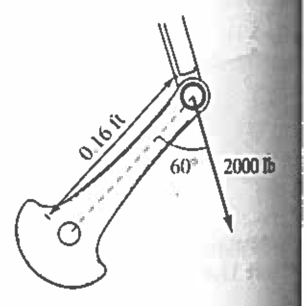
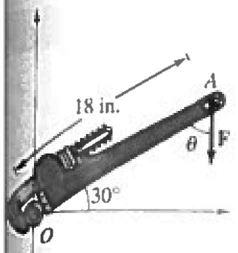


Figure for 38

38. ***Torque*** Both the magnitude and the direction of the force on a crankshaft change as the crankshaft rotates. Find the torque on the crankshaft using the position and data shown in the figure.

39. ***Optimization*** A force of 60 pounds acts on the pipe wrench shown in the figure on the next page.
    (a) Find the magnitude of the moment about O by evaluating $\|\overrightarrow{OA} \times \mathbf{F}\|$. Use a graphing utility to graph the resulting function of θ.
    (b) Use the result of part (a) to determine the magnitude of the moment when $\theta = 45°$.
    (c) Use the result of part (a) to determine the angle θ when the magnitude of the moment is maximum. Is the answer wh[at] you expected? Why or why not?



gure for 39

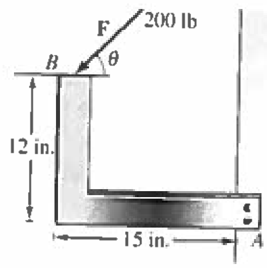


Figure for 40

Optimization A force of 200 pounds acts on the bracket shown in the figure.

(a) Determine the vector $\overrightarrow{AB}$ and the vector **F** representing the force. (**F** will be in terms of θ.)

(b) Find the magnitude of the moment about A by evaluating $\|\overrightarrow{AB} \times \mathbf{F}\|$.

(c) Use the result of part (b) to determine the magnitude of the moment when $\theta = 30°$.

(d) Use the result of part (b) to determine the angle θ when the magnitude of the moment is maximum. At that angle, what is the relationship between the vectors **F** and $\overrightarrow{AB}$? Is it what you expected? Why or why not?

(e) Use a graphing utility to graph the function for the magnitude of the moment about A for $0° \le \theta \le 180°$. Find the zero of the function in the given domain. Interpret the meaning of the zero in the context of the problem.

xercises 41–44, find u · (v × w).

$\mathbf{u} = \mathbf{i}$
$\mathbf{v} = \mathbf{j}$
$\mathbf{w} = \mathbf{k}$

42. $\mathbf{u} = \langle 1, 1, 1 \rangle$
$\mathbf{v} = \langle 2, 1, 0 \rangle$
$\mathbf{w} = \langle 0, 0, 1 \rangle$

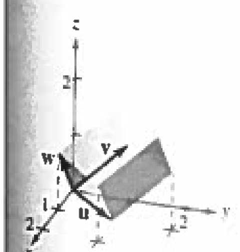
$\mathbf{u} = \langle 2, 0, 1 \rangle$
$\mathbf{v} = \langle 0, 3, 0 \rangle$
$\mathbf{w} = \langle 0, 0, 1 \rangle$

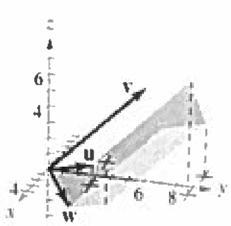
44. $\mathbf{u} = \langle 2, 0, 0 \rangle$
$\mathbf{v} = \langle 1, 1, 1 \rangle$
$\mathbf{w} = \langle 0, 2, 2 \rangle$

In Exercises 45 and 46, use the triple scalar product to the volume of the parallelepiped having adjacent edges u, d w.

$\mathbf{u} = \mathbf{i} + \mathbf{j}$
$\mathbf{v} = \mathbf{j} + \mathbf{k}$
$\mathbf{w} = \mathbf{i} + \mathbf{k}$

46. $\mathbf{u} = \langle 1, 3, 1 \rangle$
$\mathbf{v} = \langle 0, 6, 6 \rangle$
$\mathbf{w} = \langle -4, 0, -4 \rangle$





Volume **In Exercises 47 and 48, find the volume of the parallelepiped with the given vertices (see figures).**

47. $(0, 0, 0), (3, 0, 0), (0, 5, 1), (3, 5, 1)$
$(2, 0, 5), (5, 0, 5), (2, 5, 6), (5, 5, 6)$

48. $(0, 0, 0), (1, 1, 0), (1, 0, 2), (0, 1, 1)$
$(2, 1, 2), (1, 1, 3), (1, 2, 1), (2, 2, 3)$

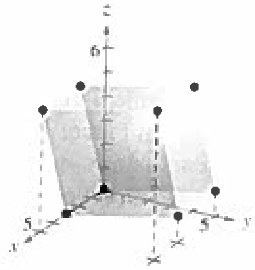


Figure for 47

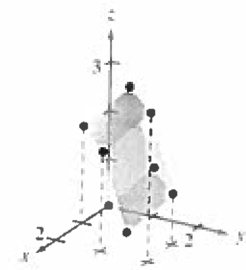


Figure for 48

Writing About Concepts

49. Define the cross product of vectors **u** and **v**.

50. State the geometric properties of the cross product.

51. If the magnitudes of two vectors are doubled, how will the magnitude of the cross product of the vectors change? Explain.

52. The vertices of a triangle in space are (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3). Explain how to find a vector perpendicular to the triangle.

True or False? **In Exercises 53–55, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

53. It is possible to find the cross product of two vectors in a two-dimensional coordinate system.

54. If $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

55. If $\mathbf{u} \neq \mathbf{0}$, $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

56. Prove Theorem 11.9.

In Exercises 57–62, prove the property of the cross product.

57. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

58. $c(\mathbf{u} \times \mathbf{v}) = (c\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (c\mathbf{v})$

59. $\mathbf{u} \times \mathbf{u} = \mathbf{0}$

60. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

61. $\mathbf{u} \times \mathbf{v}$ is orthogonal to both **u** and **v**.

62. $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if **u** and **v** are scalar multiples of each other.

63. Prove $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\|$ if **u** and **v** are orthogonal.

64. Prove $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$.