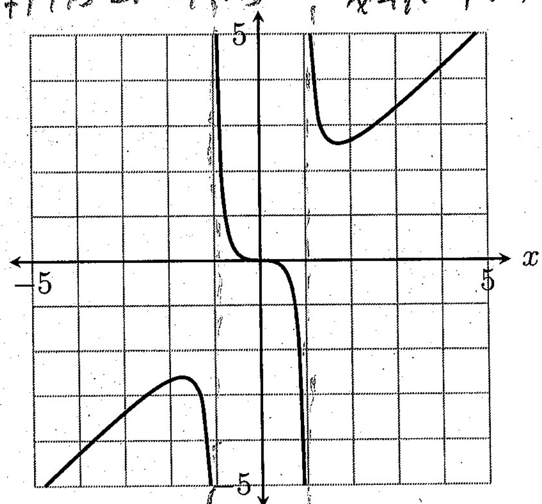
1. Below is a graph of $y = \dfrac{x^3}{x^2-1}$. Find all asymptotes, sketch them on the graph, and label the behavior near the asymptotes using the appropriate limit notation.

$\lim_{x \to -1^+} f(x)$ DNE $(+\infty)$ $\lim_{x \to 1^+} f(x)$ DNE $(+\infty)$

$N=3 \quad D=2$

No H.A.



+10

$x^2-1=0$

$x=\pm 1$ VA

$\lim_{x \to -1^-} f(x)$ DNE $(-\infty)$ $\lim_{x \to 1^-} f(x)$ DNE $(-\infty)$

2. Find $\displaystyle\lim_{x\to-\infty} \frac{1-x^2}{e^x-1}$.

Form: $\dfrac{-\infty}{-1}$ DNE $(+\infty)$

+9

3. Find $\displaystyle\lim_{x\to\infty} \frac{1-x^2}{e^x-1}$.

Form: $\dfrac{\infty}{\infty}$

$\overset{LR}{=} \lim_{x\to\infty} \dfrac{-2x}{e^x} \overset{LR}{=} \lim_{x\to\infty} \dfrac{-2}{e^x} = 0$

+9

4. Find $\displaystyle\lim_{x\to\infty}\frac{x^3-x}{16-2x^3}$.

Rational function $N = D = 3$

$$= \frac{1}{-2} = -\frac{1}{2}$$

+9

5. Find $\displaystyle\lim_{x\to 2}\frac{x^3-x}{16-2x^3}$.

Form $\frac{6}{0}$ DNE

+9

6. Find $\displaystyle\frac{d}{dx}3^{2x+1}$.

$$\left(3^{2x+1}\ln 3\right)2$$

$$= 2\cdot 3^{2x+1}\ln 3$$

+8

7. Find $\displaystyle\frac{d}{dx}x\log_2(x)$.

Product Rule

$f(x) = x \qquad f'(x) = 1$

+8

$g(x) = \log_2(x) \qquad g'(x) = \frac{1}{x\ln 2}$

$f(x)\,g'(x) + f'(x)\,g(x)$

$$\frac{x}{x\ln 2} + \log_2(x)$$

8. Find $\lim_{x\to\infty}(x+3)e^{-x}$.

$= \lim_{x\to\infty} \frac{x+3}{e^x} \overset{LR}{=} \lim_{x\to\infty} \frac{1}{e^x} = 0$

+9

9. Find $\lim_{x\to 0^+} x\ln(x)$.

$= \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{x}} \overset{LR}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$

$= \lim_{x\to 0^+} (-x) = 0$

+9

10. Given the curve described by the equation $x^3 - 3xy = y^2 + 1$, find $\frac{dy}{dx}$.

$\frac{d}{dx} x^3 - \frac{d}{dx} 3xy = \frac{d}{dx} y^2 + \frac{d}{dx} 1$

$3x^2 - 3(x\frac{dy}{dx} + y) = 2y\frac{dy}{dx}$

+10

$3x^2 - 3x\frac{dy}{dx} - 3y = 2y\frac{dy}{dx}$

$3x^2 - 3y = 3x\frac{dy}{dx} + 2y\frac{dy}{dx}$

$3x^2 - 3y = (3x+2y)\frac{dy}{dx}$

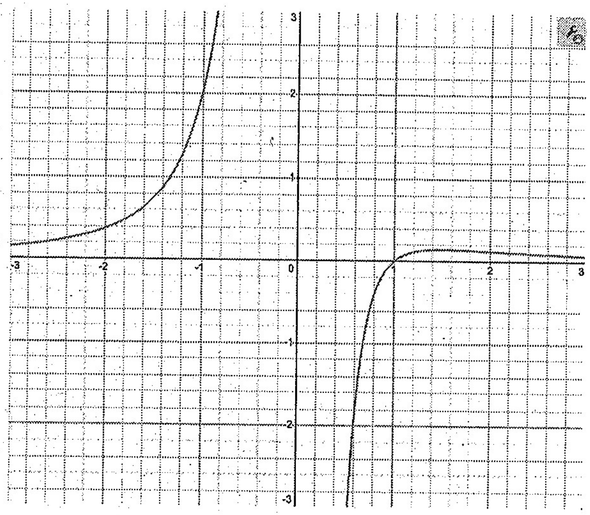
$\frac{dy}{dx} = \frac{3x^2 - 3y}{3x+2y}$

11. Consider $f(x) = \dfrac{x-1}{x^3}$, where $f'(x) = \dfrac{3-2x}{x^4}$ and $f''(x) = \dfrac{6(x-2)}{x^5}$.

(a) Find all inflection points.

(b) By creating a **sign chart,** find the intervals where $f(x)$ is concave up and concave down.

+10



$$f''(x) = \frac{6(x-2)}{x^5} = 0$$

$$6(x-2) = 0$$

$$x = 2$$

CU CD CU

+ − +

0 2

↑

for VA

$$f(2) = \frac{2-1}{8} = \frac{1}{7}$$

Inflection pt. at $(2, \frac{1}{7})$

CU: $(-\infty, 0) \cup (2, \infty)$

CD: $(0, 2)$