1. (10) Find $\lim_{x \to -\infty} \dfrac{x^2+1}{e^x}$. Of the form $\dfrac{\infty}{0}$.

DNE $(+\infty)$

2. (10) Find $\dfrac{d}{dx} 5^{4-2x}$.

$f(x) = 5^x \qquad f'(x) = 5^x \ln 5$

$g(x) = 4-2x \qquad g'(x) = -2$

$$f'(g(x))\,g'(x) = 5^{g(x)} \ln 5 \, (-2)$$

$$= -2 \ln 5 \left(5^{4-2x}\right)$$

3. (10) Find $\dfrac{d}{dx} \log_4(1+x^3)$.

$f(x) = \log_4(x) \qquad f'(x) = \dfrac{1}{x \ln 4}$

$g(x) = 1+x^3 \qquad g'(x) = 3x^2$

$$f'(g(x))\,g'(x) = \frac{1}{g(x) \ln 4} \cdot 3x^2$$

$$= \frac{3x^2}{\ln 4 \, (1+x^3)}$$

4. (10) Find $\lim_{x\to 1^-} \frac{x^2+1}{x-1}$.

Of the form $\frac{2}{0}$, so DNE.

Since $x-1<0$ as $x\to 1^-$, DNE($-\infty$)

5. (10) Find $\lim_{x\to\infty} \frac{\ln(x)}{x^2}$.

Of the form $\frac{\infty}{\infty}$

$$\overset{LR}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$$

6. (15) Find $\frac{dy}{dx}$ if $2x^2 - xy - y = 4$.

$$\frac{d}{dx}(2x^2) - \frac{d}{dx}(xy) - \frac{d}{dx}y = \frac{d}{dx}4$$

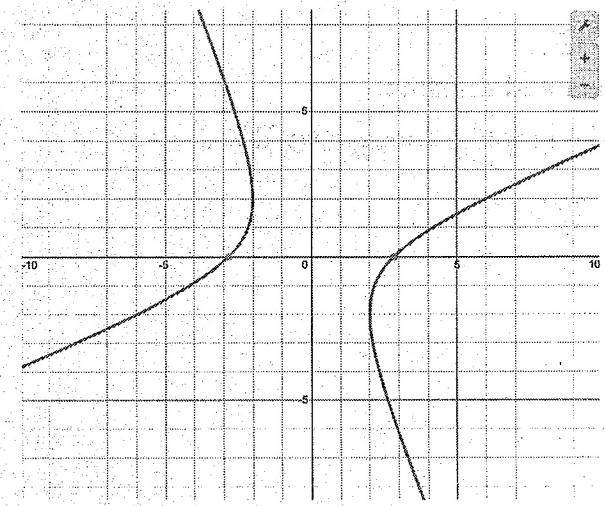
$$4x - x\frac{dy}{dx} - y - \frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} - \frac{dy}{dx} = y - 4x$$

$$\frac{dy}{dx}(-x-1) = y-4x$$

$$\frac{dy}{dx} = \frac{y-4x}{-x-1} = \frac{4x-y}{x+1}$$

7. (15) Consider the hyperbola $x^2 - 2xy - y^2 = 8$. Using calculus, (1) show that there are no horizontal tangents, and (2) find the points where there are vertical tangents. You are given that $\frac{dy}{dx} = \frac{x-y}{x+y}$.



(1) $\frac{dy}{dx} = \frac{x-y}{x+y} = 0 \Rightarrow x - y = 0 \Rightarrow x = y$

Sub. in: $x^2 - 2x(x) - x^2 = 8$

$-2x^2 = 8$ impossible

(2) $\frac{dy}{dx} = \frac{x-y}{x+y}$. Set denom $= 0$.

$x + y = 0$

$y = -x$

$x^2 - 2x(-x) - (-x)^2 = 8$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2, \; y = -x$

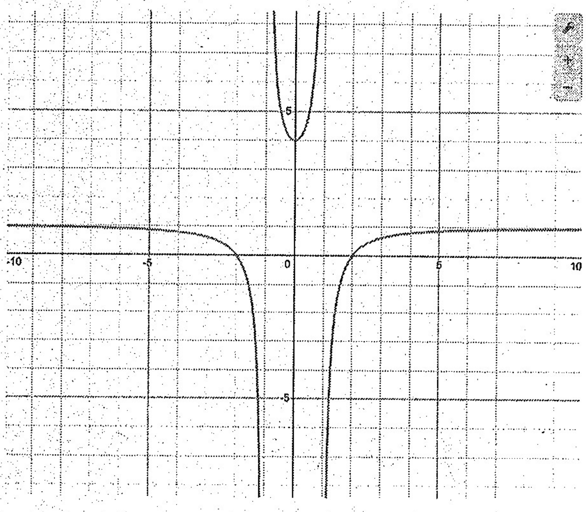
Vertical tangents at $(-2, 2)$ and $(2, -2)$

8. (20) Consider the graph of $f(x) = \dfrac{x^2-4}{x^2-1}$. You are given that $f'(x) = \dfrac{6x}{(x^2-1)^2}$, and $f''(x) = -\dfrac{6(3x^2+1)}{(x^2-1)^3}$.

5 (a) Determine any horizontal asymptotes.

7 (b) Using calculus, find all local minima and maxima.

8 (c) Using calculus, determine where the graph is concave up/down.



(a) $N=2$, $D=2$, so H.A. at ratio of leading coefficients: $y = \frac{1}{1} = 1$

(b) $f'(x) = \dfrac{6x}{(x^2-1)^2} = 0 \Rightarrow x = 0$

$f''(0) = 6 > 0$, so a local min at $(0, 4)$

(c) $f''(x)$ is never 0. Use V.A. to make sign chart

$x^2 - 1 = 0 \Rightarrow x = \pm 1$

$-$ $+$ $-$

-1 1

Concave up on $(-1, 1)$

Concave down on $(-\infty, -1) \cup (1, \infty)$