1. If $f(x) = \sin(x) + \ln(x)$, find $f''(x)$.

$f'(x) = \cos(x) + \frac{1}{x}$

$f''(x) = -\sin(x) - \frac{1}{x^2}$

2. Suppose a population of bacteria is modeled by $P(t) = 6000e^{0.03t}$, where P is the population at time t, which is given in hours. At what rate is the population increasing at 5 hours?

$P'(t) = 6000\, e^{0.03t}\,(.03) = 180\, e^{0.03t}$

$P'(5) = 180 \cdot e^{0.03 \cdot 5} \approx 209.13$

210 bacteria / hour

3. If $h(x) = \ln(x + e^x)$, find $h'(x)$.

Chain Rule

$f'(g(x))\, g'(x)$

$f(x) = \ln(x) \quad f'(x) = \frac{1}{x}$

$g(x) = x + e^x \quad g'(x) = 1 + e^x$

$\frac{1}{g(x)}(1+e^x)$

$\frac{1+e^x}{x+e^x}$

4. Find $\frac{d}{dx} e^{\cos(x)}$.

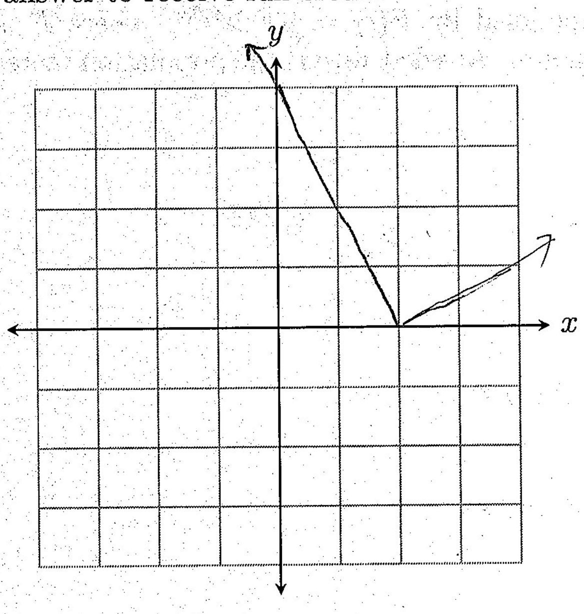
$e^{\cos(x)} \cdot (-\sin(x))$

$-\sin(x)\, e^{\cos(x)}$

5. Consider the following piecewise-defined function. Assume b is a constant.

$$g(x) = \begin{cases} \frac{1}{2}x - 1, & x < 2, \\ b - 2x, & x \geq 2. \end{cases} \quad 4 - 2x$$

What must be the value of b so that $g(x)$ is a continuous function? Sketch a graph of this function on the interval $[-4, 4]$ below. You *must* use limits correctly in your answer to receive full credit.



$\lim_{x \to 2^-} g(x) = \frac{1}{2} \cdot 2 - 1 = 0$ $\lim_{x \to 2^+} g(x) = b - 2 \cdot 2 = b - 4$

To be continuous at $x = 2$, these limits must be equal.

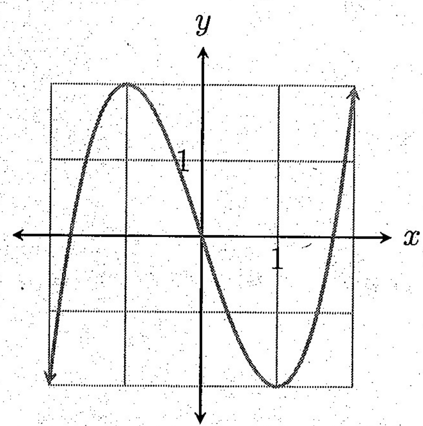
$0 = b - 4$

$b = 4$

6. Simplify $\ln(e^{2x})$.

$2x$

7. Find the local extrema for the function $f(x) = x^3 - 3x$. You are given that $f'(x) = 3x^2 - 3$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



$f'(x) = 3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$f''(x) = 6x$

$f''(-1) = -6 < 0$

Local max at $(-1, 2)$

$f''(1) = 6 > 0$

Local min at $(1, -2)$

8. Begin with a positive number. Take the square root, then multiply by 2. Subtract 4. Then subtract three times the original number. What is the largest possible result? Set up with an appropriate function and closed interval, but do not solve. Set up ONLY.

x : positive number

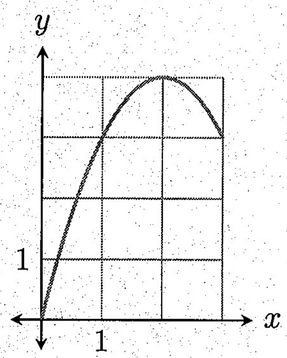
$f(x) = 2\sqrt{x} - 4 - 3x$

$[0, 4]$

x	$f(x)$
0	-4
1	-5
4	-8

↓ Keeps getting smaller

9. Find the global extrema for the function $f(x) = 4x - x^2$ on the closed interval $[0, 3]$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



$f'(x) = 4 - 2x = 0$

$2x = 4$

$x = 2$

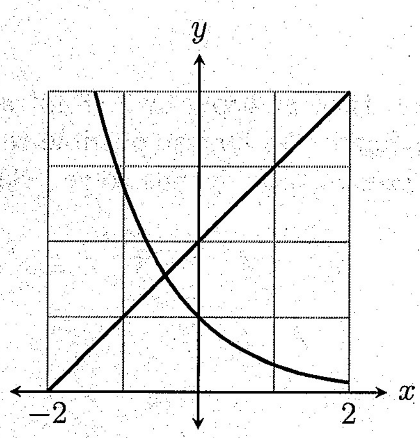
$f'(x)$ is defined everywhere on $[0,3]$.

$f(0) = 0$ ← global min at $(0,0)$

$f(2) = 4$ ← global max at $(2,4)$

$f(3) = 3$

10. Show that the graphs of $f(x) = e^{-x}$ and $f(x) = x + 2$ intersect somewhere in the interval $[-2, 2]$.



Use $h(x) = e^{-x} - x - 2$

$h(-2) = e^{2} - (-2) - 2 \approx 7.4$

$h(2) = e^{-2} - 2 - 2 \approx -3.9$

Since 0 is between -3.9 and 7.4, there is x_0 in $(-2,2)$ such that $h(x_0) = 0$. The curves intersect at x_0.