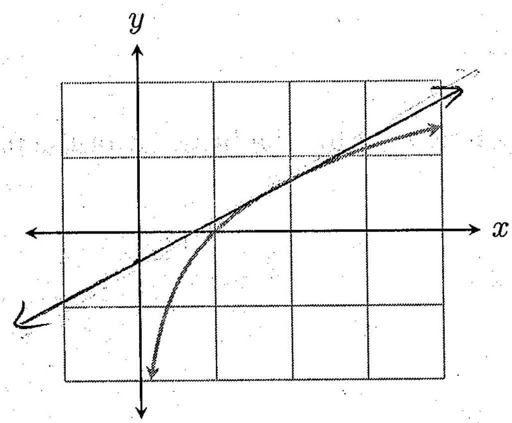
1. (10) Suppose a population of bacteria is modeled by $P(t) = 5000e^{0.02t}$, where P is the population at time t, which is given in hours. At what rate is the population increasing at 3 hours?

$$P'(t) = 5000e^{0.02t}(0.02)$$
$$= 100e^{0.02t}$$

$$P'(3) = 100e^{0.06} \approx 107 \text{ bacteria/hr}$$

2. (10) Find the equation of the tangent line to $y = \ln(x)$ at $x = 2$. You may round numbers to two decimal places.



$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2} = \text{slope}$

point $= (2, \ln(2)) \approx (2, 0.69)$

$y - 0.69 = \frac{1}{2}(x-2)$

$= \frac{1}{2}x - 1$

$y = \frac{1}{2}x - 0.31$

3. (10) Find $\frac{d}{dx}e^{x^2-x}$.

$f(x) = e^x \quad f'(x) = e^x$

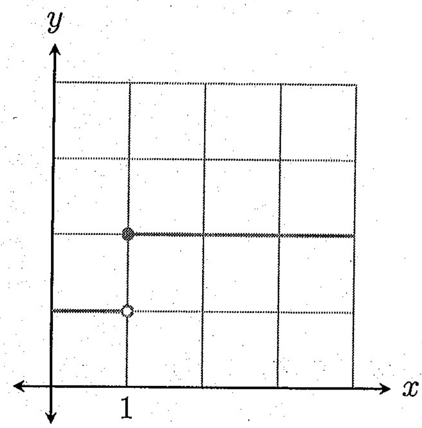
$g(x) = x^2 - x \quad g'(x) = 2x-1$

$f'(g(x))\,g'(x)$

$= e^{g(x)}(2x-1)$

$= (2x-1)e^{x^2-x}$

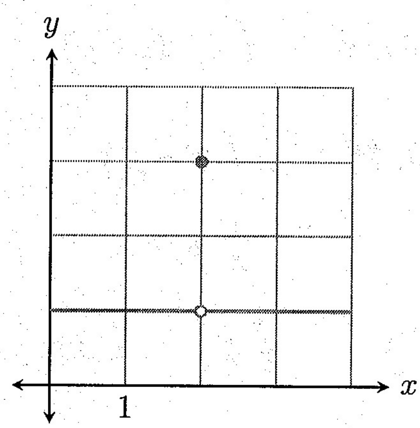
4. (9) Using notation and terminology as appropriate, describe the behavior of the function below at $x = 1$.



$\lim_{x \to 1^-} f(x) = 1$ $\lim_{x \to 1^+} f(x) = 2$

Since these limits are not equal, there is an essential discontinuity at $x = 1$

5. (9) Using notation and terminology as appropriate, describe the behavior of the function below at $x = 2$.

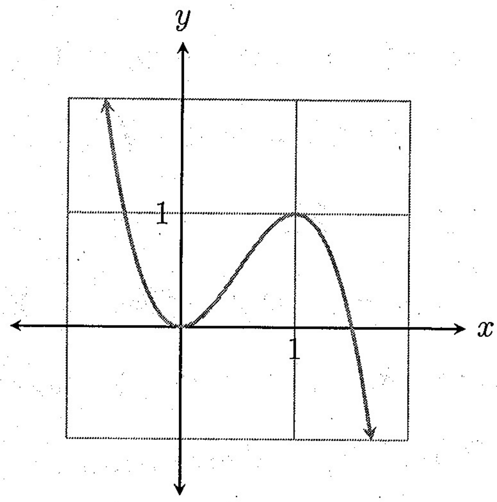


$\lim_{x \to 2^-} f(x) = 1$ $\lim_{x \to 2^+} f(x) = 1$

$f(2) = 3$.

Since the limits are equal, but not to the function value, there is a removable discontinuity at $x = 2$.

6. (15) Find the local extrema for the function $f(x) = 3x^2 - 2x^3$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



$f'(x) = 6x - 6x^2 = 0$

$6x(1-x) = 0$

$x = 0, 1$

$f''(x) = 6 - 12x$

$f''(0) = 6 - 12 \cdot 0 = 6 > 0$, so CU

Thus, a local min at $(0,0)$.

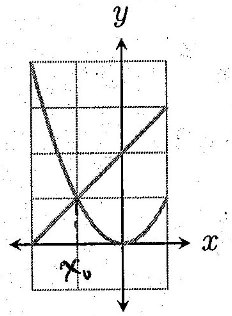
$f''(1) = 6 - 12 \cdot 1 = -6 < 0$, so CD

Thus, a local max at $(1,1)$

7. (10) Begin with a positive number. Take the reciprocal, and then add it to the original number. What is the smallest possible result? Set up with an appropriate function and closed interval, but do not solve. Set up ONLY.

$f(x) = x + \frac{1}{x}$, $[0.1, 10]$

8. (12) Show that the curves $y = x^2$ and $y = x + 2$ intersect on the interval $[-2, 1]$. You *must* use the appropriate theorem from calculus, you *cannot* just look at the graph. It is there as a guide only.



$f(x) = x^2 - (x+2) = x^2 - x - 2$

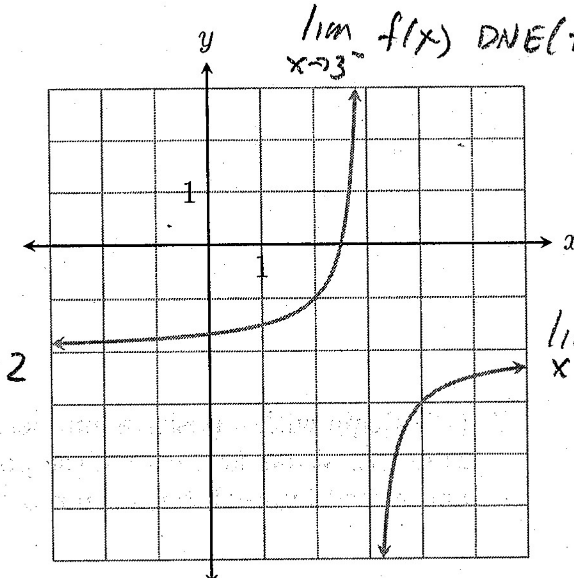
$f(-2) = (-2)^2 - (-2) - 2 = 4$

$f(1) = 1^2 - 1 - 2 = -2.$

Since $-2 < 0 < 4$, by the IVT there is some x_0 in $(-2, 1)$ with $f(x_0) = 0$. This x_0 determines a point where the graphs intersect.

9. (15) The graph of $f(x) = \dfrac{5-2x}{x-3}$ is shown to the right. Determine the asymptotes using calculus and describe the behavior at the asymptotes by neatly writing the appropriate limits on the graph.

$\lim\limits_{x \to 3^-} f(x)$ DNE $(+\infty)$



$\lim\limits_{x \to -\infty} f(x) = -2$

$\lim\limits_{x \to \infty} f(x) = -2$

$\lim\limits_{x \to 3^+} f(x)$ DNE $(-\infty)$

$N = 1$

$D = 1$

So H.A. at $y = \frac{-2}{1} = -2$

V.A. when $x - 3 = 0$

$x = 3$