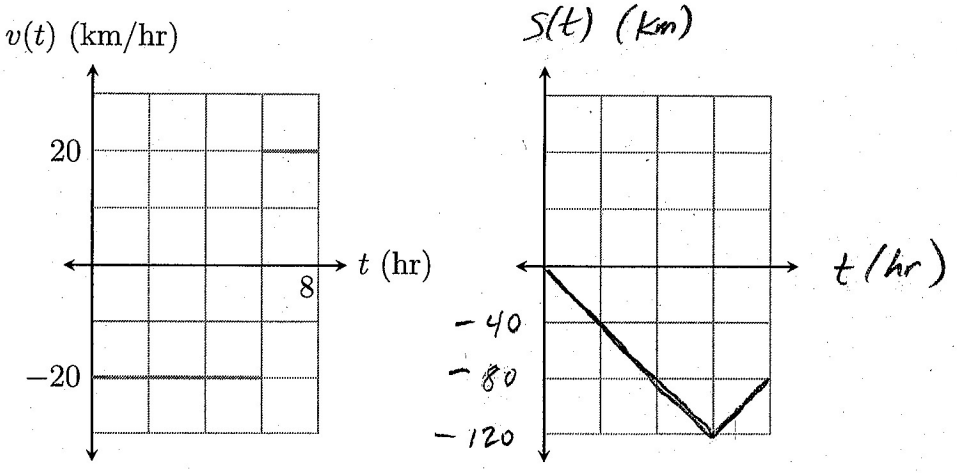
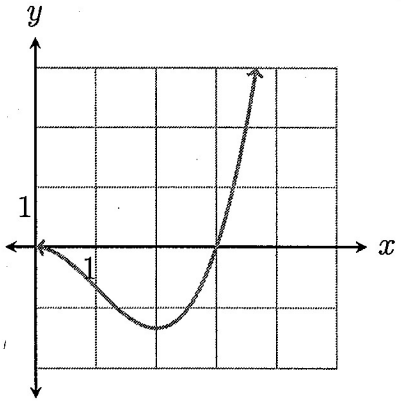
1. You are given a velocity graph below. Draw the corresponding displacement graph on the blank grid. Label axes carefully!



Write a brief sentence describing this journey.

2. Below is a graph of the function $f(x) = \frac{1}{3}x^3 - x^2$. Find an equation of the tangent line in the form $y = mx + b$ at $x = 3$. You can use the graph to verify your answer, but you have to use calculus to find the equation.



3. Find the derivatives of the following functions.

   (a) $h(x) = -\dfrac{5}{x^4}$

   (b) $h(x) = x^2 \sin(x)$

   (c) $h(x) = \dfrac{x^2}{\cos(x)}$

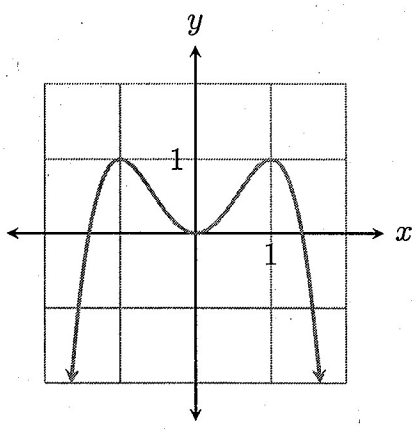
   (d) $h(x) = \tan(1 - x^2)$

4. Suppose a population of bacteria is modeled by $P(t) = 4000e^{0.02t}$, where P is the population at time t, which is given in hours. At what rate is the population increasing at 7 hours?

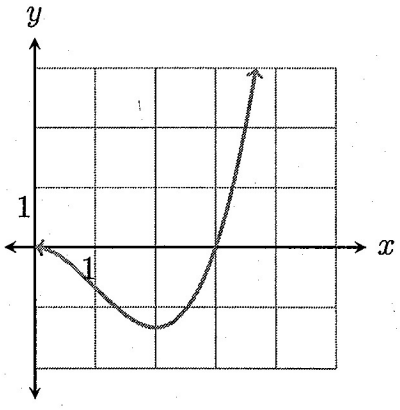
5. (a) If $h(x) = \ln(x^2 + \cos(x))$, find $h'(x)$.

   (b) Find $\dfrac{d}{dx} 3e^{(x^2+1)}$.

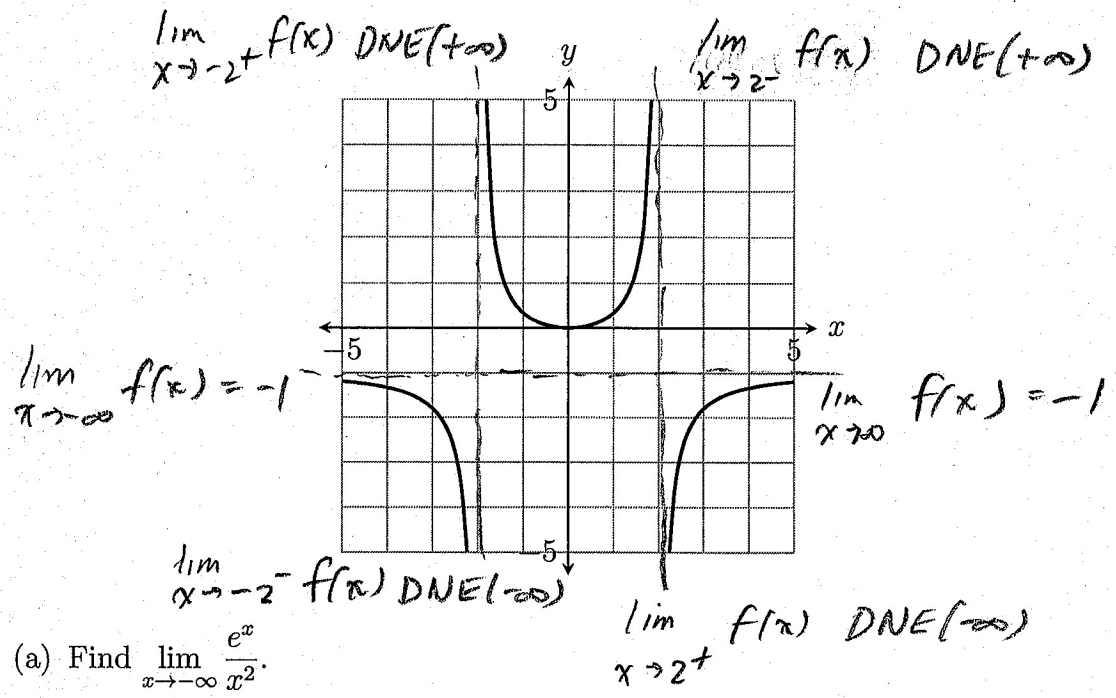
6. Find the local extrema for the function $f(x) = 2x^2 - x^4$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



7. Find the global extrema for the function $f(x) = \dfrac{1}{3}x^3 - x^2$ on the closed interval $[0, 5]$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



8. Below is a graph of $y = \dfrac{x^2}{4 - x^2}$. Find all asymptotes, sketch them on the graph, and label the behavior near the asymptotes using the appropriate limit notation.



9. (a) Find $\lim\limits_{x \to -\infty} \dfrac{e^x}{x^2}$.

   (b) Find $\lim\limits_{x \to \infty} \dfrac{e^x}{x^2}$.

10. Given the curve described by the equation $e^{xy} = x^3 + y^3$, find $\dfrac{dy}{dx}$.

SOLUTIONS, FE PRACTICE

1. You travel west at 20 km/hr for 6 hours, then you travel east at 20 km/hr for 2 hours.

2. $f'(x) = \frac{1}{3}(3x^2) - 2x = x^2 - 2x$

slope: $f'(3) = 3^2 - 2 \cdot 3 = 3$

point: $f(3) = \frac{1}{3}(3^3) - 3^2 = 0$ $\quad (3,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 3)$

$y = 3x - 9$

3. a) $h(x) = -5x^{-4}$

$h'(x) = -5(-4x^{-5}) = 20x^{-5} = \frac{20}{x^5}$

b) $f(x) = x^2 \qquad f'(x) = 2x$

$g(x) = \sin(x) \qquad g'(x) = \cos(x)$

$f(x)g'(x) + g(x)f'(x)$

$x^2 \cdot \cos(x) + \sin(x) \cdot 2x$

$x^2\cos(x) + 2x\sin(x)$

c) $f(x) = x^2 \qquad f'(x) = 2x$

$g(x) = \cos(x) \qquad g'(x) = -\sin(x)$

$$\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$\frac{\cos(x) \cdot 2x - x^2(-\sin(x))}{(\cos(x))^2} = \frac{2x\cos(x) + x^2\sin(x)}{\cos^2(x)}$$

3d) $f(x) = \tan(x)$ $\quad f'(x) = \sec^2(x)$

$g(x) = 1 - x^2$ $\quad g'(x) = -2x$

$f'(g(x))\, g'(x)$

$\sec^2(g(x))\,(-2x)$

$-2x \sec^2(1 - x^2)$

4. $P'(t) = 4000\,(e^{0.02t})(.02) = 80\,e^{0.02t}$

$P'(7) = 80\,e^{0.02 \cdot 7} \approx 92$ bacteria / hr.

5a) $f(x) = \ln(x)$ $\quad f'(x) = \frac{1}{x}$

$g(x) = x^2 + \cos(x)$ $\quad g'(x) = 2x - \sin(x)$

$f'(g(x))\, g'(x)$

$\frac{1}{g(x)}\,(2x - \sin(x))$

$\frac{2x - \sin(x)}{x^2 + \cos(x)}$

b) $f(x) = 3e^x$ $\quad f'(x) = 3e^x$

$g(x) = x^2 + 1$ $\quad g'(x) = 2x$

$f'(g(x))\, g'(x)$

$3e^{g(x)} \cdot 2x$

$6x\,e^{x^2+1}$

6. $f'(x) = 4x - 4x^3 = 0$

$4x(1-x^2) = 0$

$x = 0, \quad x = \pm 1$

$f''(x) = 4 - 12x^2$

$f''(-1) = 4 - 12(-1)^2 = -8 < 0$

local max at $(-1, 1)$

$f''(0) = 4 - 12 \cdot 0^2 = 4 > 0$

local min at $(0, 0)$

$f''(1) = 4 - 12(1^2) = -8 < 0$

local max at $(1, 1)$

7. $f'(x) = \frac{1}{3} \cdot 3x^2 - 2x = x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

$f(0) = \frac{1}{3} \cdot 0^3 - 0 = 0$

$f(2) = \frac{1}{3} \cdot 2^3 - 2^2 = -\frac{4}{3}$ $\quad (2, -\frac{4}{3})$ global min

$f(5) = \frac{1}{3} \cdot 5^3 - 5^2 = \frac{50}{3}$ $\quad (5, \frac{50}{3})$ global max

8. $N = 2, \quad D = 2, \quad \frac{1}{-1} = -1$ H.A $y = -1$

$4 - x^2 = 0$

$x^2 = 4$

$x = 2, \quad x = -2$ V.A.

9a) $\lim_{x \to -\infty} \frac{e^x}{x^2}$ of the form $\frac{0}{\infty}$. The limit is 0

b) $\lim_{x \to \infty} \frac{e^x}{x^2}$ of the form $\frac{\infty}{\infty}$

$\overset{LR}{=} \lim_{x \to \infty} \frac{e^x}{2x} \overset{LR}{=} \lim_{x \to \infty} \frac{e^x}{2}$ DNE $(+\infty)$

10. $\frac{d}{dx} e^{xy} = \frac{d}{dx} x^3 + \frac{d}{dx} y^3$

$e^{xy}\left(x \frac{dy}{dx} + y\right) = 3x^2 + 3y^2 \frac{dy}{dx}$

$x e^{xy} \frac{dy}{dx} + y e^{xy} = 3x^2 + 3y^2 \frac{dy}{dx}$

$x e^{xy} \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 3x^2 - y e^{xy}$

$\left(x e^{xy} - 3y^2\right) \frac{dy}{dx} = 3x^2 - y e^{xy}$

$\frac{dy}{dx} = \frac{3x^2 - y e^{xy}}{x e^{xy} - 3y^2}$