1. Suppose a population of bacteria is modeled by

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$$P(t) = 3000e^{0.01t},$$

where P is the population at time t, which is given in hours.

+1 (a) What is the initial population?

+2 (b) What is the population after 4 hours?

+4 (c) At what rate is the population increasing at 4 hours?

a) 3000

b) $P(4) \approx 3123$ bacteria

c) $P'(t) = 3000e^{0.01t}(.01) = 30e^{0.01t}$

$P'(4) \approx 32$ bacteria per hour

+6 2. Find $\frac{d}{dx}\ln(x^2+x)$.

$f(x) = \ln x$ $\quad f'(x) = \frac{1}{x}$

$g(x) = x^2+x$ $\quad g'(x) = 2x+1$

$f'(g(x))\,g'(x)$

$\frac{1}{g(x)}(2x+1) = \frac{2x+1}{x^2+x}$

+6 3. Find $\frac{d}{dx}x^2e^x$.

$f(x) = x^2$ $\quad f'(x) = 2x$

$g(x) = e^x$ $\quad g'(x) = e^x$

$f(x)g'(x) + g(x)f'(x)$

$x^2 \cdot e^x + e^x \cdot 2x$

$x^2e^x + 2xe^x$

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+7

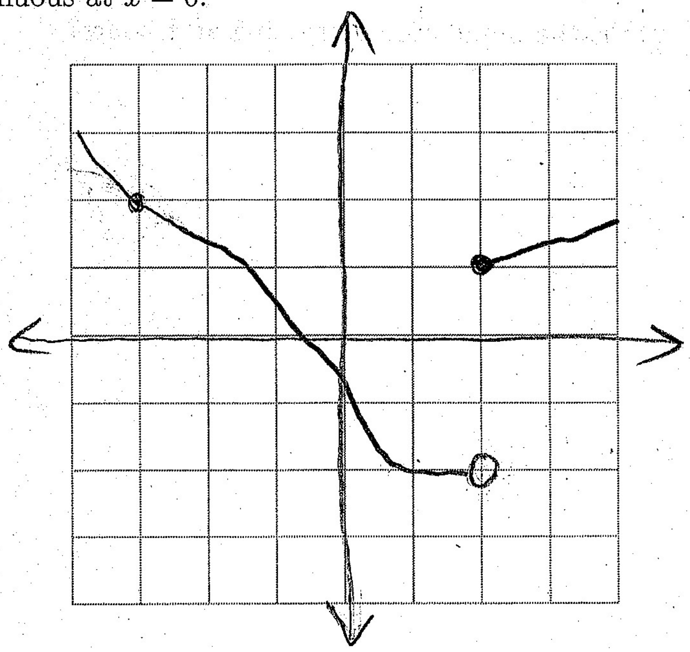
4. On the grid below, sketch a graph of a function $f(x)$ which has the following properties. Note: many answers are possible; there is not just one correct answer.

   (a) There is an essential discontinuity at $x = 2$.

   (b) $\lim_{x \to 2^-} f(x) = -2$.

   (c) $f(-3) = 2$.

   (d) $f(x)$ is continuous at $x = 0$.



+4

5. Suppose a function $f(x)$ is defined for all real numbers. You know that

$$\lim_{x \to 4^-} f(x) = 0, \quad \lim_{x \to 4^+} f(x) = -1.$$

   Which of the following are possible? Circle all that apply.

   (a) $f(x)$ is continuous at $x = 4$.

   (b) $f(x)$ has an essential discontinuity at $x = 4$.

   (c) $f(x)$ has a removable discontinuity at $x = 4$.

   (d) $f(4)$ is undefined.

6. Suppose $f(x)$ is defined for all real numbers. You know that $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 7$. If $f(x)$ is continuous at $x = 7$, what else do you know must be true?

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