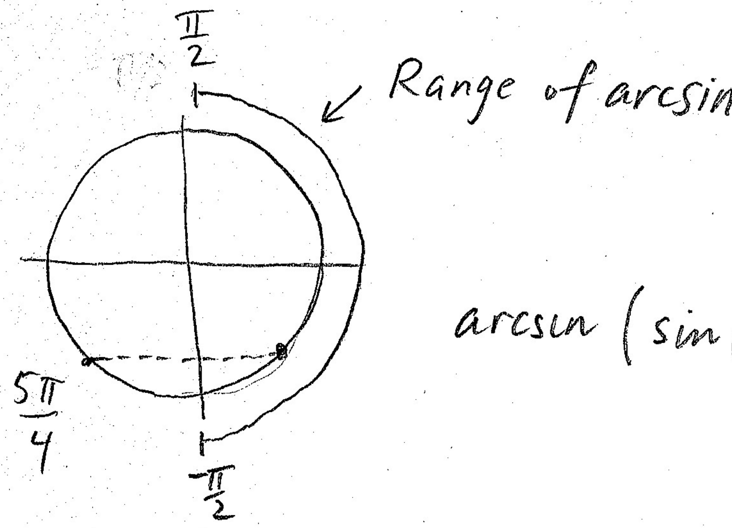
SOLUTIONS
EXAM 2

1.



$$\arcsin\left(\sin\left(\frac{5\pi}{4}\right)\right) = -\frac{\pi}{4}$$

2. $\ln\left(\frac{1}{e}\right) = \ln\left(e^{-1}\right) = -1$

3. $f(x) = \ln x \qquad g(x) = x^3 - 2x + \tan(x)$

$f'(x) = \frac{1}{x} \qquad g'(x) = 3x^2 - 2 + \sec^2(x)$

$$f'(g(x))\, g'(x) = \frac{3x^2 - 2 + \sec^2(x)}{x^3 - 2x + \tan(x)}$$

4. $f(x) = \arcsin(x) \qquad g(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$f'(x) = \frac{1}{\sqrt{1-x^2}} \qquad g'(x) = 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

$$f'(g(x))\, g'(x) = \frac{1}{\sqrt{1-(2\sqrt{x})^2}} \cdot \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{1-4x}\,\sqrt{x}}$$

5. $$\lim_{x\to\infty} \underbrace{\frac{x^2 - 10x + 5}{3e^x}}_{\frac{\infty}{\infty}} \overset{LR}{=} \lim_{x\to\infty} \underbrace{\frac{2x - 10}{3e^x}}_{\frac{\infty}{\infty}} \overset{LR}{=} \lim_{x\to\infty} \frac{2}{3e^x} = 0$$

6. 1) Evaluate f at the critical numbers.

$$f'(x) = 4x - 4x^3 = 4x(1-x^2) = 4x(1+x)(1-x)$$

$$x = 0, -1, 1$$

$\uparrow$ not in the domain [0, 2]

$$f(0) = 2\cdot 0^2 - 0^4 = 0$$

$$f(-1) = 2(-1)^2 - (-1)^4 = 1$$

2) Evaluate f at the endpoints.

$$f(-2) = 2(-2)^2 - (-2)^4 = 8 - 16 = -8$$

$$f(0) = 0$$

3) Choose max/min from 1), 2)

Absolute max at $(-1, 1)$

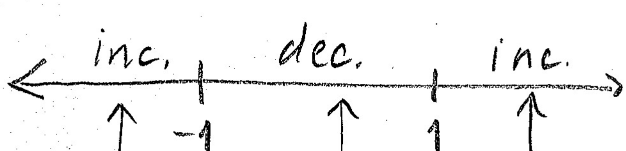
Absolute min at $(-2, -8)$.

7. $f(x) = x^3 - 3x$

$$f'(x) = 3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$3(x+1)(x-1) = 0 \Rightarrow x = -1, 1$$



test pt	test pt	test pt.
$x = -2$	$x = 0$	$x = 2$
$3(-2)^2 - 3$	$3(0)^2 - 3$	$3(2)^2 - 3$
$= 9 > 0$	$= -3 < 0$	$= 9 > 0$

Increasing on $(-\infty, -1)$ and $(1, \infty)$, decreasing on $(-1, 1)$.

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8. $f(x) = x^2 - x + 2e^x$

$f'(x) = 2x - 1 + 2e^x$

$f''(x) = 2 + 2e^x > 0$ for all x.

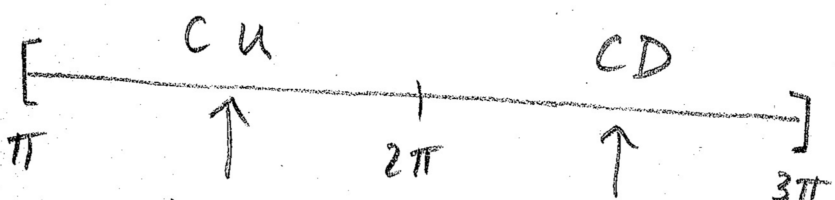
Since $f''(x) > 0$ for all x, $f(x)$ is concave up on the interval $(-\infty, \infty)$.

9. $y = \sin(x)$

$y' = \cos(x)$

$y'' = -\sin(x) = 0$ on the interval $[\pi, 3\pi]$.

$x = \pi, 2\pi, 3\pi$.



test pt. $x = \frac{3\pi}{2}$ $\qquad$ test pt. $x = \frac{5\pi}{2}$

$-\sin\left(\frac{3\pi}{2}\right) = 1$ $\qquad$ $-\sin\left(\frac{5\pi}{2}\right) = -1$

Thus there is an inflection at $x = 2\pi$.