SOLUTIONS, EXAM 2

+5 1. $\cos(3\pi) = -1$.

$\arccos(-1) = \pi$ since the range of arccos is $[0,\pi]$

+5 2. $\ln(e^2) = 2$ since $\ln x$ and e^x are inverse functions.

+10 3. Let $f(x) = e^x$ $\quad g(x) = 2x + 3\sin(x)$

$f'(x) = e^x$ $\quad g'(x) = 2 + 3\cos(x)$

$$\frac{d}{dx}\left(e^{2x+3\sin(x)}\right) = f'(g(x))g'(x)$$

$$= (2+3\cos(x))\, e^{2x+3\sin(x)}$$

+10 4. Let $f(x) = \arctan(x)$ $\quad g(x) = \sqrt{x}$

$f'(x) = \dfrac{1}{x^2+1}$ $\quad g'(x) = \dfrac{1}{2\sqrt{x}}$

$$\frac{d}{dx}\left(\arctan(\sqrt{x})\right) = f'(g(x))\, g'(x)$$

$$= \frac{1}{\sqrt{x}^2+1} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{(x+1)(2\sqrt{x})}$$

+15 5. $$\lim_{x\to 0} \underbrace{\frac{\cos(x)-1}{x^2}}_{\frac{0}{0}} \overset{LR}{=} \lim_{x\to 0} \underbrace{\frac{-\sin(x)}{2x}}_{\frac{0}{0}} \overset{LR}{=} \lim_{x\to 0} \frac{-\cos(x)}{2} = -\frac{1}{2}$$

OR $$= \lim_{x\to 0} \left(-\frac{1}{2}\right)\underbrace{\left(\frac{\sin(x)}{x}\right)}_{=1} = -\frac{1}{2}\cdot 1 = -\frac{1}{2}$$

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SOLUTIONS, EXAM 2

6. $f(x) = 3x - x^3$

1) Evaluate f at critical points.

$$f'(x) = 3 - 3x^2 = 0$$
$$3 = 3x^2$$
$$x^2 = 1$$
$x = 1$ (note: $x = -1$ is not in $[0,2]$)

$f(1) = 3(1) - 1^3 = 2$

This is the only critical point, since $f'(x)$ is defined for all x.

2) Evaluate f at the endpoints

$f(0) = 3 \cdot 0 - 0^3 = 0$

$f(2) = 3 \cdot 2 - 2^3 = 6 - 8 = -2$

3) Look for largest/smallest from 1) and 2)

Absolute max: $f(1) = 2$

Absolute min: $f(2) = -2$

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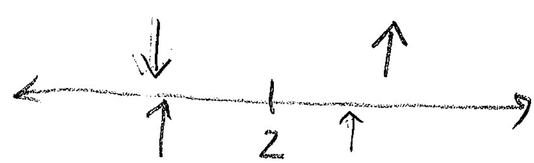
7. $f(x) = x^2 - 4x + 5$

$f'(x) = 2x - 4 = 0$

$2x = 4$

$x = 2$



$x=1$. test point

$f'(1) = 2(1) - 4 = -2 < 0$

$x=3$. test point

$f'(3) = 2 \cdot 3 - 4 = 2 > 0$

Increasing on $(2, \infty)$.

Decreasing on $(-\infty, 2)$.

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8. $f(x) = \ln(x)$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -x^{-2} = -\frac{1}{x^2}$

The domain of $\ln(x)$ is $(0,\infty)$. For all values of x in $(0,\infty)$, $-\frac{1}{x^2}$ is negative, which means that $\ln(x)$ is concave down.

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9. $f(x) = \cos(x)$

$f'(x) = -\sin(x)$

$f''(x) = -\cos(x) = 0$

$\cos(x) = 0$ on $[0, 2\pi]$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Test points: $\frac{\pi}{4}$ $\quad$ π $\quad$ $\frac{7\pi}{4}$

$-\cos\left(\frac{\pi}{4}\right) < 0$ $\quad$ $-\cos(\pi) = 1$ $\quad$ $-\cos\left(\frac{7\pi}{4}\right) < 0$

CD $\quad$ CU $\quad$ CD

Since concavity changes, $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are inflection points.