We are working in the integers, denoted by $\mathbb{Z}$.

We say a *divides* b and write "$a \mid b$" if there is an integer k such that $ka = b$.

If $a \mid b$ and $a \mid c$, then $a \mid (b+c)$, $a \mid (b-c)$, and $a \mid (bc)$. We do not have a result for division (yet) since a/b might not be an integer.

We say $a \pmod{m} = r$ if $a = qm + r$ where $0 \leq r < m$. q, r, are the quotient and remainder. m is a positive integer at least 2.

We write $a \equiv b \pmod{m}$ if $a \pmod{m} = b \pmod{m}$, or equivalently, $m \mid (b-a)$.

If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then:

1. $a + c \equiv b + d \pmod{m}$,
2. $a - c \equiv b - d \pmod{m}$, and
3. $ac \equiv bd \pmod{m}$.

Now for your homework problems!

1. Decide if the following are true or false.
   (a) $3 \mid 12$.
   (b) $12 \mid 3$.
   (c) $7 \mid -14$.
   (d) $10 \mid 10$.
   (e) $-1 \mid -1$.
   (f) $-1 \mid 17$.
2. Explain why $4 \mid 0$, but $0 \nmid 4$.
3. Suppose you know that $a \mid (b+c)$. Can you conclude that $a \mid b$ and $a \mid c$? Explain your answer.

4. $a = 72$, and $b = 2^m 3^n$, where $m, n > 0$. If $a \mid b$, what can you conclude about m and n?

5. Find the following.

   (a) 23 (mod 7).

   (b) -23 (mod 7).

   (c) 12 (mod 12).

   (d) 1,000,000 (mod 11).

   (e) $(-3)^{97}$ (mod 4).

   (f) $(10k - 3)$ (mod 5), where k is any integer.

   (g) $91p$ (mod 7), where p is any integer.

6. Show, using arithmetic (mod 10), than if n is any positive integer, then n and n^5 end in the same digit.

7. Find a number n such that $n \equiv 2 \pmod 3$ and $n \equiv 2 \pmod 7$. Can you find *every* number n with this property?