

1. For each of the following, circle YES if L'Hôpital's Rule can be applied, and NO if it cannot.

(a) YES NO $\lim_{x \rightarrow \infty} \frac{\ln(x)}{e^{-x}}$

(b) YES NO $\lim_{x \rightarrow -\infty} \frac{x}{2^x}$

(c) YES NO $\lim_{x \rightarrow 0} \frac{\sin(x)}{1 - \cos(x)}$

(d) YES NO $\lim_{x \rightarrow \infty} \frac{e^x}{1 + x^2}$

2. Use L'Hôpital's Rule to evaluate.

(a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1} \stackrel{LR}{=} \lim_{x \rightarrow 1} \frac{2x}{3x^2} = \lim_{x \rightarrow 1} \frac{2}{3x} = \frac{2}{3}$

(b) $\lim_{x \rightarrow -\infty} \frac{e^{-x}}{x^2 + 1} \stackrel{LR}{=} \lim_{x \rightarrow -\infty} \frac{-e^{-x}}{2x}$
 $\stackrel{LR}{=} \lim_{x \rightarrow -\infty} \frac{e^{-x}}{2} = \text{DNE} (+\infty)$

(c) $\lim_{x \rightarrow \infty} \frac{x}{2^x} \stackrel{LR}{=} \lim_{x \rightarrow \infty} \frac{1}{2^x \ln 2} = 0$

3. Find $\lim_{x \rightarrow \infty} x^2 e^{-2x}$.

$$= \lim_{x \rightarrow \infty} \frac{x^2}{e^{2x}} \quad \text{LR} \quad = \lim_{x \rightarrow \infty} \frac{2x}{2e^{2x}}$$

$$\text{LR} \quad = \lim_{x \rightarrow \infty} \frac{2}{4e^{2x}} = 0$$

4. Find $\lim_{x \rightarrow -\infty} \frac{5^x}{2^x}$.

$$= \lim_{x \rightarrow -\infty} \left(\frac{5}{2}\right)^x = 0$$

5. Find $\frac{d}{dx} 3^{1-2x}$.

$$f(x) = 3^x$$

$$f'(x) = 3^x \ln 3$$

$$g(x) = 1-2x$$

$$g'(x) = -2$$

$$\begin{aligned} f'(g(x)) g'(x) &= 3^{g(x)} \ln 3 (-2) \\ &= -2 \ln 3 \cdot 3^{1-2x} \end{aligned}$$

6. Find $\frac{d}{dx} \log_3(1+x^3)$.

$$f(x) = \log_3(x)$$

$$f'(x) = \frac{1}{x \ln 3}$$

$$g(x) = 1+x^3$$

$$g'(x) = 3x^2$$

$$\begin{aligned} f'(g(x)) g'(x) &= \frac{1}{g(x) \ln 3} \cdot 3x^2 \\ &= \frac{3x^2}{(\ln 3)(1+x^3)} \end{aligned}$$