

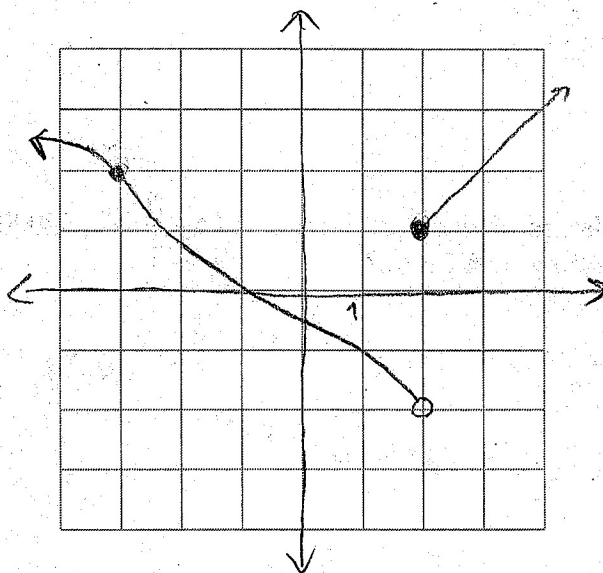
1. On the grid below, sketch a graph of a function $f(x)$ which has the following properties.
Note: many answers are possible; there is not just one correct answer.

(a) There is an essential discontinuity at $x = 2$.

(b) $\lim_{x \rightarrow 2^-} f(x) = -2$.

(c) $f(-3) = 2$.

(d) $f(x)$ is continuous at $x = 0$.



2. If $h(x) = e^x \cos(x)$, find $h'(x)$.

$f(x) = e^x$

$f'(x) = e^x$

$g(x) = \cos(x)$

$g'(x) = -\sin(x)$

$$\begin{aligned} f(x)g'(x) + g(x)f'(x) &= e^x(-\sin(x)) + \cos(x) \cdot e^x \\ &= -e^x \sin(x) + e^x \cos(x) \end{aligned}$$

3. If $h(x) = x^3 \ln(x)$, find $h'(x)$.

$f(x) = x^3$

$f'(x) = 3x^2$

$g(x) = \ln(x)$

$g'(x) = \frac{1}{x}$

$$\begin{aligned} f(x)g'(x) + g(x)f'(x) &= x^3 \cdot \frac{1}{x} + \ln(x) \cdot 3x^2 \\ &= x^2 + 3x^2 \ln(x) \end{aligned}$$

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4. Suppose a function $f(x)$ is defined on all real numbers. You are given that there is an essential discontinuity at $x = 3$. Describe this feature of the graph using appropriate limit notation.

$$\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$$

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5. Suppose a population of bacteria is modeled by $P(t) = 8000e^{0.01t}$. At what rate is the population increasing at 4 hours?

$$\begin{aligned} P'(t) &= 8000 e^{0.01t} (0.01) \\ &= 80 e^{0.01t} \end{aligned}$$

$$P'(4) = 80 e^{0.01(4)} \approx 83.3$$

Increasing at 84 bacteria/hr.

- +6
6. We see from the graph of $y = \ln(x)$ that this function is increasing on its domain. Show this using calculus.

See text.