1. (10) Find $\lim_{x\to-\infty} \frac{x^4+1}{e^x}$.

Of the form $\frac{\infty}{0}$

DNE $(+\infty)$

2. (10) Find $\frac{d}{dx}7^{3-4x}$.

$f(x) = 7^x$ $f'(x) = 7^x \ln 7$

$g(x) = 3-4x$ $g'(x) = -4$

$f'(g(x))g'(x) = 7^{g(x)} \ln 7 \, (-4)$

$= -4 \ln 7 \cdot 7^{3-4x}$

3. (10) Find $\frac{d}{dx}\log_5(1+x^4)$.

$f(x) = \log_5(x)$ $f'(x) = \frac{1}{x \ln 5}$

$g(x) = 1 + x^4$ $g'(x) = 4x^3$

$f'(g(x))g'(x) = \frac{1}{g(x)\ln 5} \cdot 4x^3$

$= \frac{4x^3}{\ln 5 \, (1+x^4)}$

4. (10) Find $\lim_{x \to 1^+} \frac{x^2+1}{1-x}$. Of the form $\frac{2}{0}$, so DNE.

Since $1-x<0$ as $x \to 1^+$, DNE$(-\infty)$

5. (10) Find $\lim_{x \to \infty} \frac{\ln(x)}{x^3}$. Of the form $\frac{\infty}{\infty}$

$$\stackrel{LR}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{3x^2} = \lim_{x \to \infty} \frac{1}{3x^3} = 0$$

6. (15) Find $\frac{dy}{dx}$ if $x^2 - xy - 2y = 4$.

$$\frac{d}{dx} x^2 - \frac{d}{dx}(xy) - \frac{d}{dx} 2y = \frac{d}{dx} 4$$

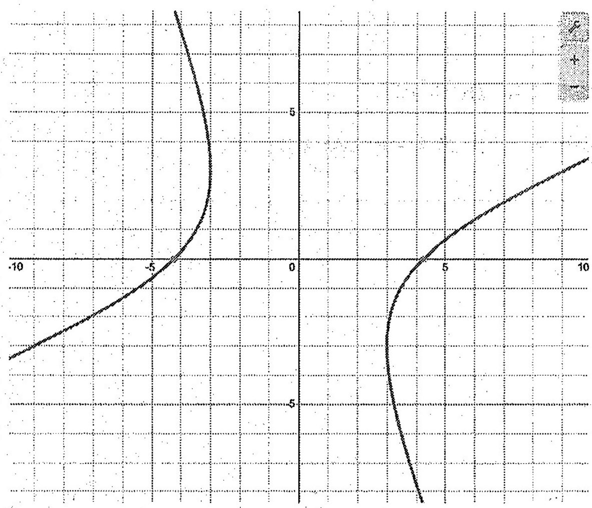
$$2x - x\frac{dy}{dx} - y - 2\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} - 2\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx}(-x-2) = y - 2x$$

$$\frac{dy}{dx} = \frac{y-2x}{-x-2} = \frac{2x-y}{x+2}$$

7. (15) Consider the hyperbola $x^2 - 2xy - y^2 = 18$. Using calculus, (1) show that there are no horizontal tangents, and (2) find the points where there are vertical tangents. You are given that $\frac{dy}{dx} = \frac{x-y}{x+y}$.



(1) $\frac{dy}{dx} = \frac{x-y}{x+y} = 0 \Rightarrow x - y = 0 \Rightarrow x = y$

Sub. in: $x^2 - 2x(x) - x^2 = 18$

$-2x^2 = 18$ impossible.

(2) $\frac{dy}{dx} = \frac{x-y}{x+y}$ set denom $= 0$.

$x + y = 0$

$y = -x$

$x^2 - 2x(-x) - (-x)^2 = 18$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$, $y = -x$

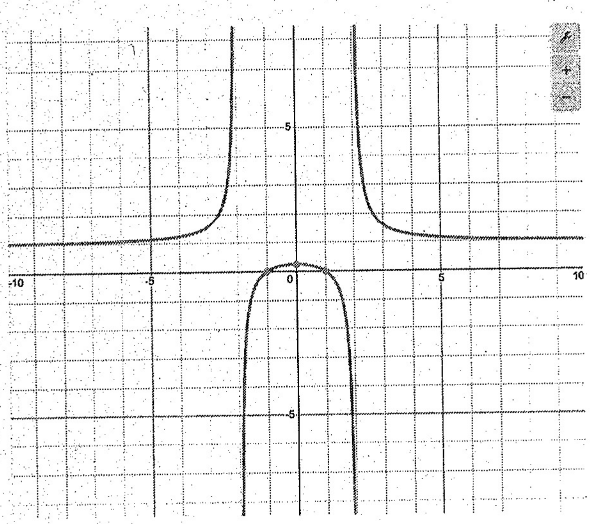
Vertical tangents at $(-3, 3)$ and $(3, -3)$

8. (20) Consider the graph of $f(x) = \frac{x^2-1}{x^2-4}$. You are given that $f'(x) = -\frac{6x}{(x^2-4)^2}$, and $f''(x) = \frac{6(3x^2+4)}{(x^2-4)^3}$.

5 (a) Determine any horizontal asymptotes.

7 (b) Using calculus, find all local minima and maxima.

8 (c) Using calculus, determine where the graph is concave up/down.



(a) $N = 2$, $D = 2$, so H.A. at ratio of leading coefficients: $y = \frac{1}{1} = 1$.

(b) $f'(x) = -\frac{6x}{(x^2-4)^2} = 0 \Rightarrow x = 0$

$f''(0) < 0$, so a local max at $(0, \frac{1}{4})$.

(c) $f''(x)$ is never 0. Use V.A. to make sign chart

$x^2 - 4 = 0 \Rightarrow x = \pm 2$

+ − +
−2 2

Concave up on $(-\infty, -2) \cup (2, \infty)$

Concave down on $(-2, 2)$.