Exam 1~~A~~ B, 26 Sep 2023

1. You are given a displacement graph below. Draw the corresponding velocity graph on the blank grid. Label axes carefully!

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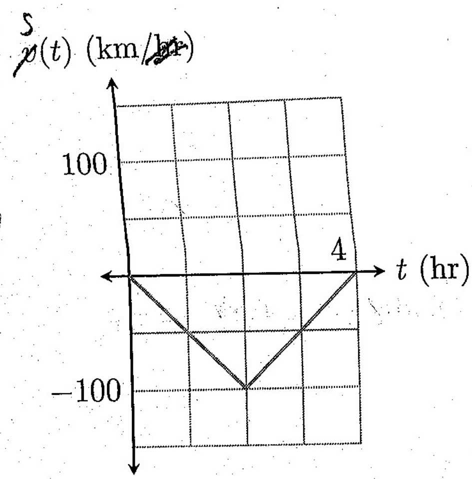
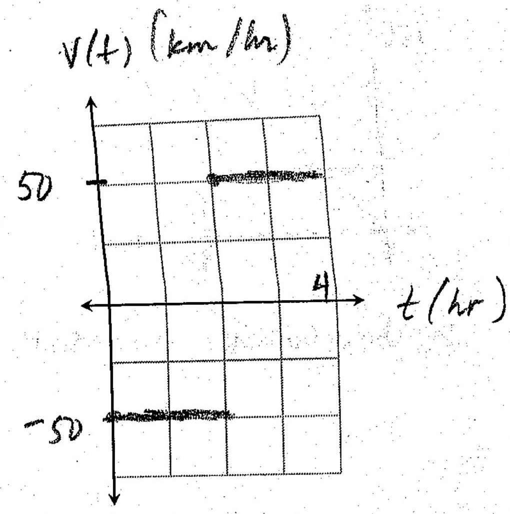
velocity is slope of $s(t)$

$\frac{-100}{2} = -50$

$\frac{100}{2} = 50$





Write a brief sentence describing this journey.

2. Below is a graph of the function $f(x) = x^3 - 2x^2$. Find an equation of the tangent line in the form $y = mx + b$ at $x = 1$. You can use the graph to verify your answer, but you have to use calculus to find the equation. You may use the fact that $f'(x) = 3x^2 - 4x$.

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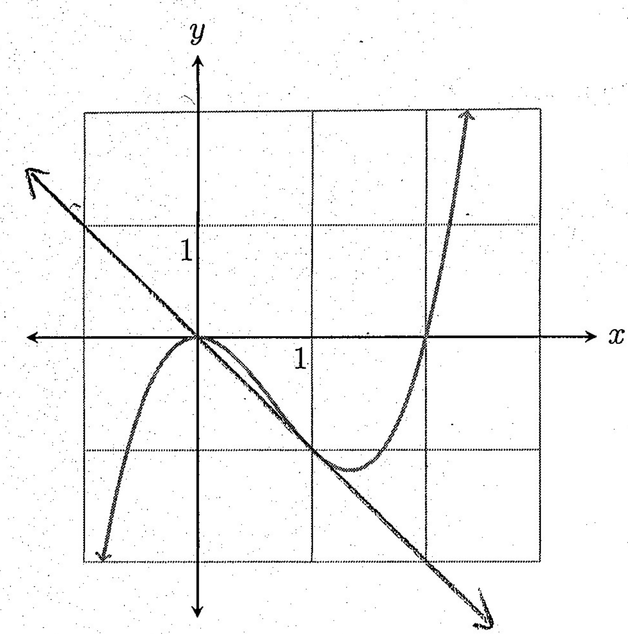
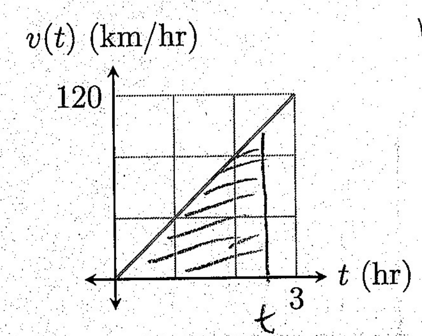


$\text{slope} = f'(1) = 3(1^2) - 4 \cdot 1$
$= -1$

$f(1) = 1^3 - 2 \cdot 1^2 = -1$

$y - (-1) = -1(x - (-1))$
$y = -x$

3. Below is a graph of a velocity curve. Find an equation for the displacement curve.



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$v(t) = \frac{120}{3}t + 0 = 40t$

$s(t) = \text{Area under } v(t)$

$= \frac{1}{2} \cdot b \cdot h$

$= \frac{1}{2} \cdot t \cdot 40t = 20t^2$

4. Using the definition of the derivative, find $f'(x)$ if $f(x) = 5 - x$.

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$$\frac{f(x+h) - f(x)}{h} = \frac{5 - (x+h) - (5-x)}{h}$$

$$= \frac{5 - x - h - 5 + x}{h} = \frac{-h}{h} = -1$$

$$\lim_{h \to 0} (-1) = -1 = f'(x)$$

5. Find the derivatives of the following functions.

(a) $h(x) = x^2\sqrt{x}$ $= x^2 \cdot x^{\frac{1}{2}} = x^{\frac{5}{2}}$

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$$h'(x) = \frac{5}{2}x^{\frac{5}{2}-1} = \frac{5}{2}x^{\frac{3}{2}}$$

(b) $h(x) = x\cos(x)$

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$f(x) = x$ $\quad f'(x) = 1$

$g(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$= x(-\sin(x)) + \cos(x) \cdot 1$

$= -x\sin(x) + \cos(x)$

+9 (c) $h(x) = \frac{\sin(x)}{x^2}$

$f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$

$g(x) = x^2$ $\quad g'(x) = 2x$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$= \frac{x^2 \cdot \cos(x) - \sin(x) \cdot 2x}{(x^2)^2}$$

$$= \frac{x^2\cos(x) - 2x\sin(x)}{x^4} = \frac{x(x\cos(x) - 2\sin(x))}{x^4}$$

$$= \frac{x\cos(x) - 2\sin(x)}{x^3}$$

(d) $h(x) = \sin(x^2 + 3x)$

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$f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$

$g(x) = x^2 + 3x$ $\quad g'(x) = 2x + 3$

$$f'(g(x))\,g'(x) = \cos(g(x))(2x+3)$$

$$= (2x+3)\cos(x^2+3x)$$

6. Suppose $f(x) = \sin(x) - x^4$. Find $f''(x)$.

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$$f'(x) = \cos(x) - 4x^3$$

$$f''(x) = -\sin(x) - 12x^2$$

7. Below is a graph of $f(x) = 2x^3 - x^4$. You are given that $f'(x) = 6x^2 - 4x^3$ and $f''(x) = 12x - 12x^2$. By making the appropriate sign chart, find all inflection points on this curve.

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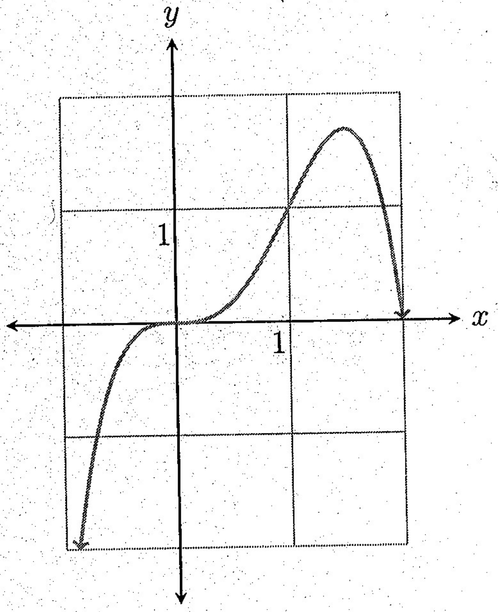

$f''(x) = 12x - 12x^2$

$= 12x(1-x) = 0$

$x = 0, \quad x = 1$

$- \quad + \quad -$

$-1 \quad 0 \quad \frac{1}{2} \quad 1 \quad 2$

Test points

$f''(-1) = 12(-1) - 12(-1)^2 = -24 < 0$

$f''(\frac{1}{2}) = 12(\frac{1}{2}) - 12(\frac{1}{2})^2 = 3 > 0$

$f''(2) = 12(2) - 12(2^2) = -24 < 0$

Concavity changes, so inflection points at (0,0) and (1,1)

For points $\begin{cases} f(0) = 2(0)^3 - 0^4 = 0 \\ f(1) = 2(1)^3 - 1^4 = 1 \end{cases}$

8. Fill in the blanks with either $f(x)$, $f'(x)$, or $f''(x)$.

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(a) To find where a function is increasing or decreasing, we use $f'(x)$.

(b) To find the y-value for a local maximum, we use $f(x)$.

(c) To make a sign chart to find inflection points, we use $f''(x)$.