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1. Suppose a population of bacteria is modeled by $P(t) = 6000e^{0.03t}$, where P is the population at time t, which is given in hours. At what rate is the population increasing at 5 hours?

$P(t) = 6000e^{0.03t}$

$P'(t) = 6000e^{0.03t}(0.03) = 180e^{0.03t}$

$P'(5) = 180e^{0.03(5)} \approx 220$ bacteria/hr.

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2. If $h(x) = \ln(x + e^x)$, find $h'(x)$.

$f(x) = \ln(x)$ $\quad f'(x) = \frac{1}{x}$

$g(x) = x + e^x$ $\quad g'(x) = 1 + e^x$

$f'(g(x))\,g'(x) = \frac{1}{g(x)}(1+e^x) = \frac{1+e^x}{x+e^x}$

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3. Find $\frac{d}{dx}e^{\cos(x)}$.

$f(x) = e^x$ $\quad f'(x) = e^x$

$g(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

$f'(g(x))\,g'(x) = e^{g(x)}(-\sin(x))$

$= -\sin(x)e^{\cos(x)}$

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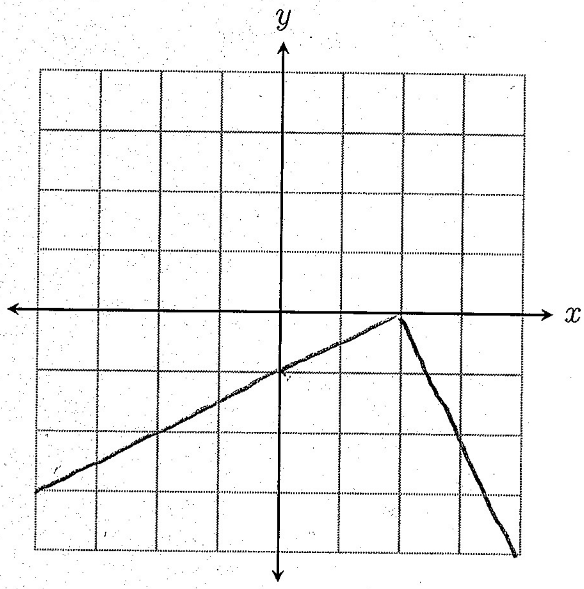
4. Consider the following piecewise-defined function. Assume b is a constant.

$$g(x) = \begin{cases} \frac{1}{2}x - 1, & x < 2, \\ b - 2x, & x \geq 2. \end{cases}$$

What must be the value of b so that $g(x)$ is a continuous function? Sketch a graph of this function on the interval $[-4, 4]$ below. You *must* use limits correctly in your answer to receive full credit.



$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} \left(\frac{1}{2}x - 1\right)$$
$$= \frac{1}{2} \cdot 2 - 1 = 0$$

$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (b - 2x)$$
$$= b - 2 \cdot 2 = b - 4$$

To be continuous at $x = 2$, these limits must be equal

$$0 = b - 4$$
$$b = 4$$

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5. Simplify $\ln(e^{2x})$. $= 2x$

6. Suppose a function $f(x)$ is defined for all real numbers. You know that

$$\lim_{x \to -3+} f(x) = f(-3).$$

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Which of the following are possible? Circle all that apply.

(a) $f(x)$ is continuous at $x = -3$.
(b) $f(x)$ has an essential discontinuity at $x = -3$.
(c) $f(x)$ has a removable discontinuity at $x = -3$.
(d) $f(x)$ is indefined at $x = -3$.

7. Find the local extrema for the function $f(x) = 3x - x^3$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.

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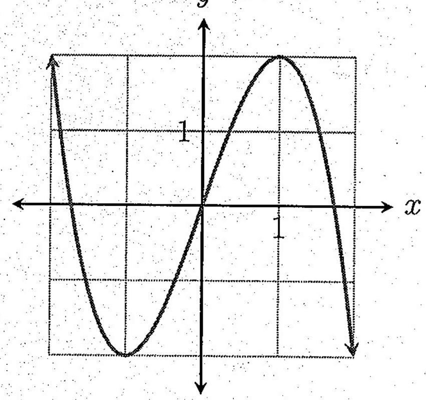

$f'(x) = 3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -6x$

$f''(-1) = -6(-1) = 6 > 0$ local min

$f''(1) = -6(1) = -6 < 0$ local max

$f(-1) = 3(-1) - (-1)^3 = -2$

$f(1) = 3(1) - 1^3 = 2$

local min: $(-1, -2)$

local max: $(1, 2)$

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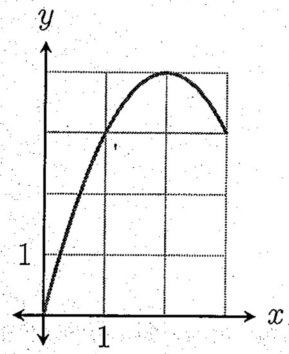
8. Find the global extrema for the function $f(x) = 4x - x^2$ on the closed interval $[0, 3]$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



$f'(x) = 4 - 2x = 0$

$2x = 4$

$x = 2$

Check values: $f(0) = 0 \leftarrow$ min

$f(2) = 4 \leftarrow$ max

$f(3) = 3$

global min: $(0,0)$

global max: $(2,4)$

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9. Begin with a positive number. Take the square root, then multiply by 2. Subtract 4. Then subtract three times the original number. What is the largest possible result? Set up with an appropriate function and closed interval, but do not solve. Set up ONLY.

$f(x) = 2\sqrt{x} - 4 - 3x$

Interval: $[0, 1]$