1. If $h(x) = \arctan(1 - x)$, find $h'(x)$.

2. Suppose you are blowing up a balloon using an air pump whose output is 4000 cm^3 of air per second (this is about 240 cubic inches). When the radius of the balloon is 15 cm, how fast is the radius expanding?

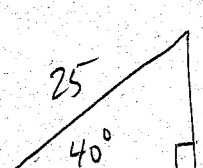
$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4000 = 4\pi \cdot 15^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 1.41 \text{ cm/s}$$

3. Suppose you throw a baseball at an angle of 40° from the horizontal at a speed of 25 m/s. When the baseball leaves your hand, it is 2 m above the ground. Write the displacement equations which describe this.



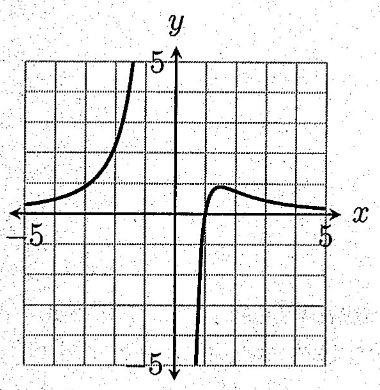
$25 \sin 40° \approx 16.07 = v_0$

$25 \cos 40° \approx 19.15 = v_h$

$$y(t) = -4.9t^2 + v_0 t + s_0$$
$$= -4.9t^2 + 16.07t + 2$$

$$x(t) = v_h t$$
$$= 19.15t$$

4. Below is a graph of $f(x) = \dfrac{6(x-1)}{x^3}$. You are given that $f'(x) = -\dfrac{6(2x-3)}{x^4}$ and $f''(x) = \dfrac{36(x-2)}{x^5}$. Find the intervals where the function is increasing and decreasing.



$f'(x) = 0 \Rightarrow 2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

$+ \qquad + \qquad -$

$-1 \quad 0 \quad 1 \quad \frac{3}{2} \quad 2$

V.A.

$f'(-1) = 30 > 0$

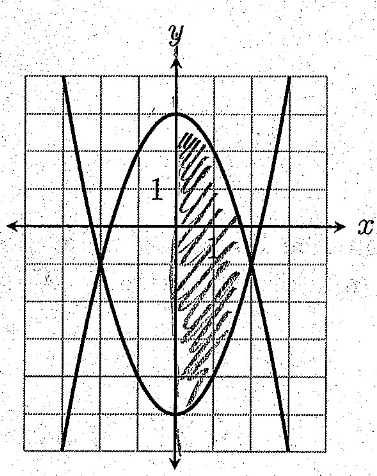
$f'(1) = 6 > 0$

$f'(2) = -\frac{3}{8} < 0$

Increasing on $(-\infty, 0) \cup (0, \frac{3}{2})$

Decreasing on $(\frac{3}{2}, \infty)$.

5. Find the area between the curves $f(x) = x^2 - 5$ and $f(x) = 3 - x^2$. The graph is for reference only; all work must be done using calculus.



Points of intersection:

$x^2 - 5 = 3 - x^2$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$\int_0^2 ((3-x^2) - (x^2-5))\, dx$

$= \int_0^2 (8 - 2x^2)\, dx = 8x - \frac{2}{3}x^3 \Big|_0^2$

$= 8 \cdot 2 - \frac{2}{3} \cdot 2^3 - (0 - 0)$

$= \frac{32}{3}$

Double this to get $\frac{64}{3}$.

6. Solve the initial value problem $f'(x) = \sin(x) - \cos(x)$, $f(\pi) = 3$.

$f(x) = \int (\sin(x) - \cos(x))\,dx = -\cos(x) - \sin(x) + C$

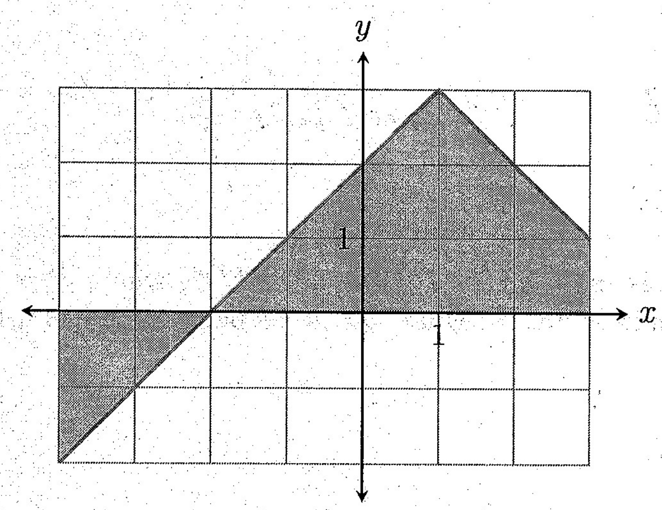
$f(\pi) = -\cos(\pi) - \sin(\pi) + C = 3$

$1 - 0 + C = 3$

$C = 2$

$f(x) = -\cos(x) - \sin(x) + 2.$

7. A graph of $f(x)$ is shown below. Find the following.



(a) $\int_{-2}^{1} f(x)\,dx$

(b) $\int_{4}^{2} f(x)\,dx$

(c) $\int_{0}^{-3} f(x)\,dx$

(d) $\int_{-2}^{-2} f(x)\,dx$

8. Find $\dfrac{d}{dx}\displaystyle\int_{3}^{x} \cos(u^2)\,du$.

~~9~~. If $\int_3^5 f(x)\,dx = -7$, what is $\int_5^3 f(x)\,dx$?

10. Find $\int x^2 \cos(1-x^3)\,dx$.

$u = 1 - x^3$

$\frac{du}{dx} = -3x^2 \Rightarrow du = -3x^2\,dx$

↑ need a "-3"

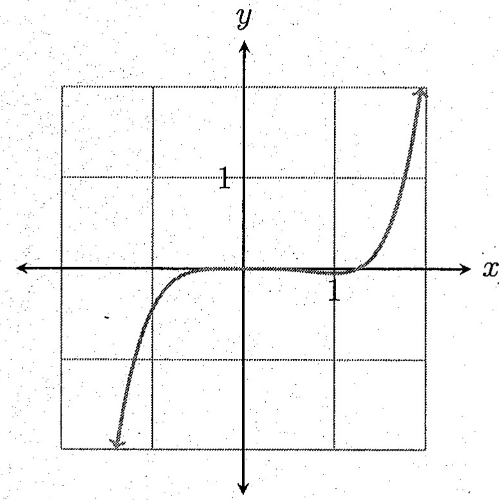
$$-\frac{1}{3}\int \overbrace{-3x^2 \cos(1-x^2)\,dx}^{du}$$

$$= -\frac{1}{3}\int \cos(u)\,du$$

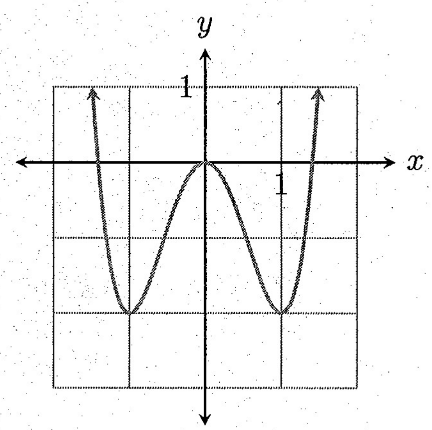
$$= -\frac{1}{3}\sin(u) + C = -\frac{1}{3}\sin(1-x^3) + C.$$

~~11~~. Suppose a population of bacteria is modeled by $P(t) = 5000e^{0.025t}$, where P is the population at time t, which is given in hours. At what rate is the population increasing at 6 hours?

~~12~~. Below is a graph of the function $f(x) = \frac{1}{5}x^5 - \frac{1}{4}x^4$. Find an equation of the tangent line in the form $y = mx + b$ at $x = -1$. You can use the graph to verify your answer, but you have to use calculus to find the equation.



13. Find the local extrema for the function $f(x) = 2x^4 - 4x^2$. You *must* show the appropriate calculus for full credit. No partial credit will be given for just looking at the graph.



$f'(x) = 8x^3 - 8x$

$f''(x) = 24x^2 - 8$

$f'(x) = 0 = 8x^3 - 8x$

$8x(x^2-1) = 0$

$x=0, \quad x = \pm 1$

$f''(0) = -8 < 0$ Local max at $(0,0)$

$f''(-1) = 24(-1)^2 - 8 = 16 > 0$ Local min at $(-1,-2)$

$f''(1) = 24 \cdot 1^2 - 8 = 16 > 0$ Local min at $(1,-2)$

14. Find the derivatives of the following functions.

(a) $h(x) = \dfrac{2}{x^6}$ $= 2x^{-6}$

$h'(x) = 2 \cdot (-6x^{-7}) = -12x^{-7} = \dfrac{-12}{x^7}$

(b) $h(x) = x^3 \cos(x)$

$f(x) = x^3$ $\quad f'(x) = 3x^2$

$g(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

$f(x)\, g'(x) + g(x)\, f'(x)$

$x^3(-\sin(x)) + \cos(x) \cdot 3x^2$

$-x^3 \sin(x) + 3x^2 \cos(x)$

(c) $h(x) = \dfrac{x}{\sin(x)}$ $\quad f(x) = x$ $\quad f'(x) = 1$

$g(x) = \sin(x)$ $\quad g'(x) = \cos(x)$

$\dfrac{g(x)\, f'(x) - f(x)\, g'(x)}{g(x)^2}$

$\dfrac{\sin(x) \cdot 1 - x \cdot \cos(x)}{(\sin(x))^2}$

$\dfrac{\sin(x) - x\cos(x)}{\sin^2(x)}$

(d) $h(x) = \cos(x^3 - x)$

$f(x) = \cos(x)$ $\quad f'(x) = -\sin(x)$

$g(x) = x^3 - x$ $\quad g'(x) = 3x^2 - 1$

$f'(g(x))\, g'(x)$

$-\sin(g(x))\,(3x^2-1)$

$-(3x^2-1)\sin(x^3-x)$

15. Below is a graph of $y = \dfrac{x}{1-x}$. Find all asymptotes, sketch them on the graph, and label the behavior near the asymptotes using the appropriate limit notation. NOTE: Make sure when you write $\lim\limits_{x \to -2^+}$, for example, the symbols are not too small. If I can't read it, points will be taken off.

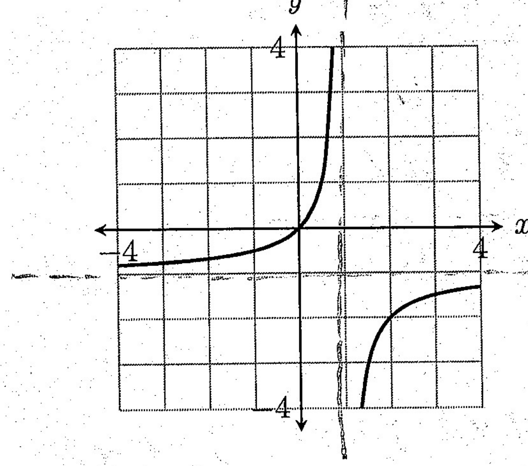
$\lim\limits_{x \to 1^-} f(x)$ DNE$(+\infty)$

$N = 1$

$D = 1$

HA at $y = \frac{1}{-1}$

$= -1$



$\lim\limits_{x \to -\infty} f(x) = -1$

$\lim\limits_{x \to \infty} f(x) = -1$

$\lim\limits_{x \to 1^+} f(x)$ DNE$(-\infty)$

V.A. Denom $= 0$

$1 - x = 0$

$x = 1$

16. Using the definition of the derivative, find $f'(x)$ if $f(x) = 3x - 2x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{3(x+h) - 2(x+h)^2 - (3x - 2x^2)}{h}$$

$$\frac{\cancel{3x} + 3h - 2(x^2 + 2xh + h^2) - \cancel{3x} + 2x^2}{h}$$

$$\frac{3h - \cancel{2x^2} - 4xh - 2h^2 + \cancel{2x^2}}{h}$$

$$\frac{3h - 4xh - 2h^2}{h}$$

$$\frac{\cancel{h}(3 - 4x - 2h)}{\cancel{h}}$$

$$\lim_{h \to 0} (3 - 4x - 2h) = 3 - 4x$$