Remember that the Final Exam will contain two question from each of the first three exams. Here are problems from the first two exams which correspond to the review we did in class today.

1. Find the equation of the line tangent to $y = x^3 + x^2$ at $x = 1$. Check your work by graphing on desmos.

2. The population of a colony of bacteria at time t (in hours) is given by the equation $P(t) = 3000e^{0.01t}$. Find out how fast the colony is growing at time $t = 3$.

3. Suppose you are given $f(x) = \frac{1}{3}x^3 - 9x$, so that $f'(x) = x^2 - 9$. Using a **sign chart,** determine where the function is increasing and decreasing. Write your answer in **interval notation.** Visually check by graphing on desmos.

4. Suppose you are given the following function and its derivatives:

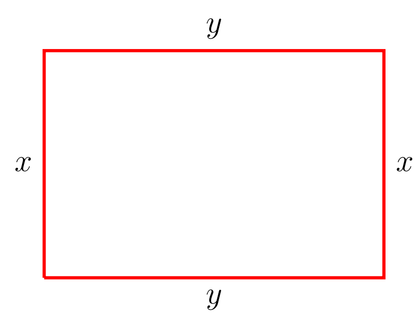
$$\begin{aligned} f(x) &= \frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x \\ f'(x) &= x^3 - 5x^2 + 3x + 9 = (x-3)^2(x+1) \\ f''(x) &= 3x^2 - 10x + 3 \end{aligned}$$

   Determine where $f'(x) = 0$. Using a **sign chart,** determine if there is a minimum, maximum, or inflection point at these values of x.

5. Find the global extrema of the function $f(x) = 4+4x-x^2$ on the closed interval $[-2, 4]$. Do *not* just make a graph and look at it. You *must* use the appropriate theorem from calculus. Visually check with desmos.

6. Show that the curves $y = x^2+2x$ and $y = 3-x-x^2$ intersect on the interval $[0, 2]$. You *must* use the appropriate theorem from calculus, you *cannot* just look at the graph. Check with desmos.

7. Suppose you want to fence in a rectangular area, as shown, and have 300 m of fencing. What is the largest area you can enclose? Set up *only.* That is, give a function $f(x)$ and a closed interval $[a, b]$. Do *not* actually work out the largest area.



8. Evaluate the following limits, using L'Hôpital's Rule when appropriate.

   (a) $\lim_{x\to\infty} x^2 \ln(2x)$

   (b) $\lim_{x\to 0^+} x^2 \ln(2x)$

Working with the Definition of the Derivative

There is usually quite a bit of algebra in working with the definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

So let's break it down into steps.

Let's start with $f(x) = x^2 - 3x$. How do we evaluate $f(x+h)$? We substitute $x+h$ everywhere we see an x. If you sometimes get stuck with this, here's something to try.

1. First, rewrite the function with boxes.

$$f\left(\boxed{}\right) = \left(\boxed{}\right)^2 - 3\left(\boxed{}\right).$$

2. Next, put $x+h$ in each empty box.

$$f\left(\boxed{x+h}\right) = \left(\boxed{x+h}\right)^2 - 3\left(\boxed{x+h}\right).$$

3. We don't need the boxes any more.

$$f(x+h) = (x+h)^2 - 3(x+h).$$

4. Expand. Be careful when distributing the minus sign.

$$f(x+h) = x^2 + 2xh + h^2 - 3x - 3h.$$

5. Now substitute into the limit definition and simplify until the h cancels. Again, watch the minus signs. Note that for the h to cancel, *every* term in the numerator that does *not* contain h should cancel.

$$\begin{aligned}
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} \\
&= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h} \\
&= \lim_{h \to 0} \frac{h(2x + h - 3)}{h} \\
&= \lim_{h \to 0} (2x + h - 3) \\
&= 2x - 3.
\end{aligned}$$

You will have a simple quadratic using the definition of the derivative on the Final Exam. It's easy to make up one on your own and try it. Make sure you use at least one negative sign, since you can be certain that I will, too.