Exam 2, MA 131M, 31 October 2022

Please show as much work as you need. Keep in mind that if you skip several steps and get an incorrect answer, it will be very difficult to assign partial credit. Please take a moment to skim through the test first and start with the questions you feel most confident about. Your answers do NOT have to be in order on your paper.

1. (15) Consider the following piecewise-defined function. Assume c is a constant.

$$g(x) = \begin{cases} cx + 4, & x \leq -1, \\ -2x, & x > -1. \end{cases}$$

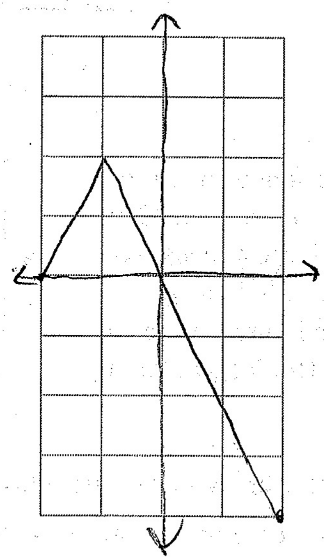
What must be the value of c so that $g(x)$ is a continuous function? Sketch a graph of this function on the interval $[-2, 2]$ below.

$\lim_{x\to -1^-} g(x) = \lim_{x \to -1^-} (cx+4)$

$= -c+4$

$\lim_{x\to -1^+} g(x) = \lim_{x \to -1^+} (-2x)$

$= 2$

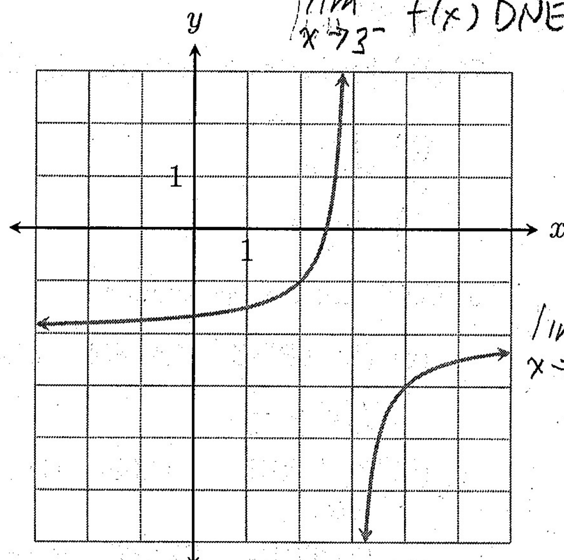


$-c+4 = 2$

$c = 2$

2. (14) The graph of $f(x) = \dfrac{5-2x}{x-3}$ is shown to the right. Describe the behavior at the asymptotes $x = 3$ and $y = -2$ by writing the appropriate limits on the graph, as done in class.

$\lim_{x\to 3^-} f(x)$ DNE $(+\infty)$



$\lim_{x\to -\infty} f(x) = -2$

$\lim_{x\to \infty} f(x) = -2$

$\lim_{x\to 3^+} f(x)$ DNE $(-\infty)$

SOLUTIONS, Exam 2

3. Since $f(x)$ is continuous (a polynomial) on a closed interval, we apply the Extreme Value Theorem.

1) $f'(x) = -2x = 0 \Rightarrow x = 0.$ $f'(x)$ is always defined.

2) $f(0) = 3,$ $f(-1) = 2,$ $f(2) = -1$

3) Global max at $(0,3)$, global min at $(2,-1)$

4. 1) $f(x) = x^2 - (x+2) = x^2 - x - 2 = 0$

2) $f(x)$ is continuos being a polynomial

3) $f(-2) = 4,$ $f(1) = -2$

4) Note that $-2 < 0 < 4.$

So, by the IVT, there is x_0 in $[-2,1]$ with $f(x_0) = 0.$

5) Thus, the curves intersect at $(x_0, f(x_0))$

5. We are looking for the largest area, so $f(x) = xy.$

200m of fencing means $2x + 2y = 200$

$$2y = 200 - 2x$$

$$y = 100 - x.$$

$f(x) = x(100-x) = 100x - x^2.$

$[0, 100]$ is a good interval since no side can be longer than 100 m.

SOLUTIONS, Exam 2

6. Since $N = 2$ and $D = 2$, then $y = \frac{4}{-1} = -4$ is a HA

Solving $x^2 - 1 = 0$

$(x+1)(x-1) = 0 \Rightarrow x = -1$ and $x = 1$ are VA.

7 (a) This limit is of the form $\frac{0}{\infty}$, so it is 0.

(b) This limit is of the form $\frac{\infty}{\infty}$, so we use LR.

$$\lim_{x \to \infty} \frac{e^x}{x} \overset{LR}{=} \lim_{x \to \infty} \frac{e^x}{1} \quad \text{DNE } (+\infty)$$

8(a) FALSE. LR is only used for limits

(b) FALSE $f(x) = \frac{x^2+1}{x}$ has no H.A.

(c) FALSE $f(x) = \frac{x}{x^2+1}$ has no VA