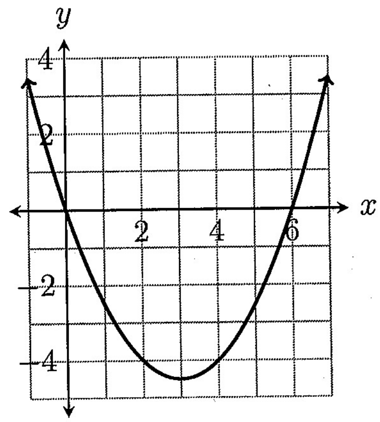
1. Let $f(x) = \frac{1}{2}x^2 - 3x$. Answer the following questions. The graph is shown below for reference only. All your answers must be justified *using calculus*, not just by looking at the graph.



(a) Find the equation of the tangent line at $x = 2$.

$f'(x) = x - 3$

$f'(2) = 2-3 = -1 \rightarrow m$

$f(2) = \frac{1}{2} \cdot 2^2 - 3 \cdot 2 = -4$

$(2,-4) = (x_1, y_1)$

$y - y_1 = m(x - x_1)$

$y - (-4) = -1(x-2)$

$y + 4 = -x + 2$

$y = -x - 2$

(b) Where is the function increasing? Write your answer in interval notation.

f is increasing when $f'(x) > 0$

$x - 3 > 0$

$x > 3$

$(3, \infty)$

(c) Where is the function decreasing? Write your answer in interval notation.

f is decreasing where $f'(x) < 0$

$x - 3 < 0$

$x < 3$

$(-\infty, 3)$

Find the derivatives of the following functions.

(a) $f(x) = -x^5 + \sqrt{x^3}$ $= -x^5 + x^{\frac{3}{2}}$

$f'(x) = -5x^4 + \frac{3}{2}x^{\frac{1}{2}}$

(b) $f(x) = \underset{f}{e^x}\,\underset{g}{\sin(x)}$

$f(x)g'(x) + g(x)f'(x)$

$e^x(\cos(x)) + \sin(x)\cdot e^x$

$e^x(\cos(x) + \sin(x))$

(c) $f(x) = \dfrac{b + x^2}{b - x^2}$ $\begin{matrix} \leftarrow f \\ \leftarrow g \end{matrix}$

$\dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$\dfrac{(b-x^2)(2x) - (b+x^2)(-2x)}{(b-x^2)^2}$

$\dfrac{4bx}{(b-x^2)^2} \leftarrow \dfrac{2bx - 2x^3 + 2bx + 2x^3}{(b-x^2)^2}$

(d) $f(x) = \ln\sqrt{1 + x^2}$ (Hint: Use rules of logarithms first.)

$= \ln(1+x^2)^{\frac{1}{2}} = \frac{1}{2}\ln(1+x^2)$

Chain rule: $f(x) = \ln x$ $f'(x) = \frac{1}{x}$

$g(x) = 1+x^2$ $g'(x) = 2x$

$\frac{1}{2}\cdot f'(g(x))\,g'(x)$

$\frac{1}{\cancel{2}}\cdot\frac{1}{g(x)}\cdot\cancel{2}x = \dfrac{x}{1+x^2}$